УДК 533.951

GUIDING CENTER DISTRIBUTION FUNCTION IN TOROIDAL MAGNETIC FIELD WITH ONE RESONANT STRUCTURE

Nikolai A. Azarenkov*, Oleg A. Shyshkin*,
Ralf Schneider, Yurii L. Igitkhanov****
**Kharkiv V.N.Karazin National University, Svobody sq. 4, 61077 Kharkiv, Ukraine*
***Max-Planck-Institut fur Plasmaphysik, EURATOM Association, Teilinstitut Greifswald, D-17489 Greifswald, Germany*

Analytical expression for the guiding center distribution function in the toroidal magnetic field of HELIAS stellarator with the one resonant structure was obtained as a solution of the drift kinetic equation. This expression gives possibility to treat analytically the critical task for the modern fusion devices, e.g. to estimate the particle transport in 3D plasma edge configurations. One resonant structure means that in a vertical cross-section of the magnetic field configuration we observe one island chain, which possesses the pure separatrix. Additional perturbations that cause additional magnetic islands and stochastic layers are not taken into account. Analytical treatment of the problem is done for the model in which the electric field is neglected. The solution of the drift kinetic equation is assumed to be the sum of $\sin\chi_1$ and $\cos\chi_1$ harmonics, where χ_1 represents the dependences on toroidal and poloidal angle variables, "wave" numbers of perturbation, perturbation frequency and time. It is shown, that in accordance with the model, the $\cos\chi_1$ harmonic vanishes and the solution, which consists of only the one $\sin\chi_1$ – harmonic, is enough to present changes in distribution function caused by a resonant structure of the magnetic filed. As a numerical example the tungsten ion guiding center distribution function is considered.
KEY WORDS: HELIAS, stellarator, guiding center equations, guiding center distribution function, magnetic field resonance, electric field potential

THE PROBLEM TO BE SOLVED

Let us consider magnetically confined plasma in a toroidal magnetic field of stellarator [1]. In such case the Larmor radius of the charged particle is much less than the plasma radius and one can describe plasma in the guiding center approximation [1, 2]. From this point of view, in a kinetic theory, one can use the distribution function of the guiding centers f^* instead of the particle distribution function f without losing the whereabouts information for the particles.

To proceed with analytical treatment of the problem we consider the model of the magnetic field in toroidal system in two steps. First of all let us imagine the case when we have the toroidal system with closed nested magnetic surfaces without any perturbations throughout the volume of confinement. That means the complete absence of any island chains. The vertical cross-section of such magnetic field surfaces is presented on the figure 1 in the flux coordinates [3].

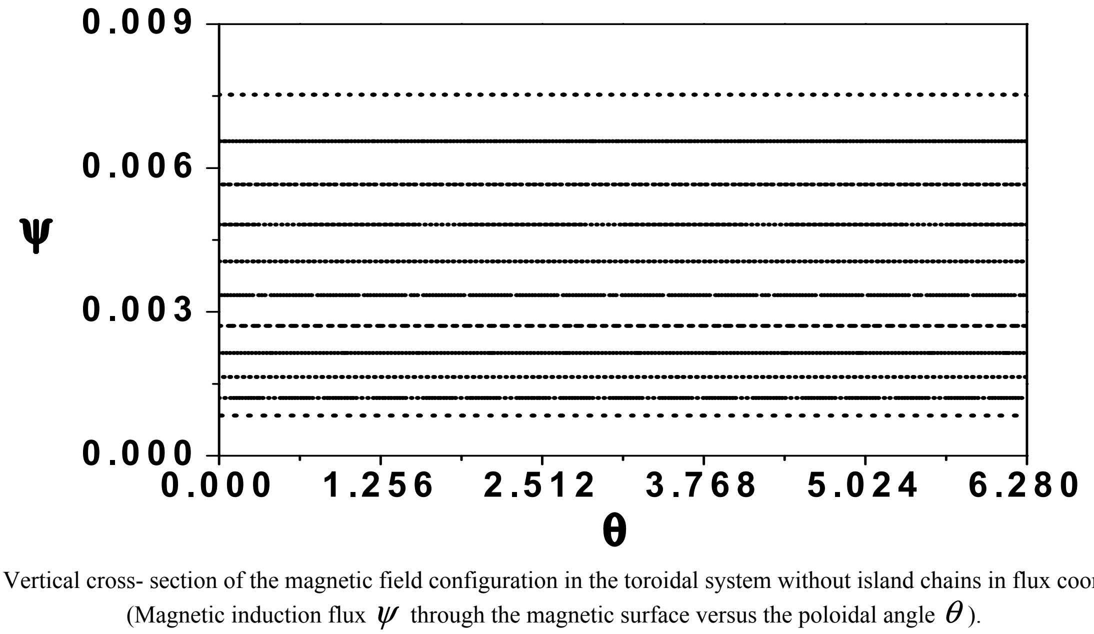

Fig.1. Vertical cross- section of the magnetic field configuration in the toroidal system without island chains in flux coordinates (Magnetic induction flux ψ through the magnetic surface versus the poloidal angle θ).

Such model seams to be in a contradiction with the realistic magnetic field configuration in stellarator devices and is always used to describe plasma features in the region of the plasma core. By including the magnetic field perturbation

[4,5,6] we create only the one island chain with the pure separatrix. The vertical cross-section of the magnetic field configuration for such case is presented on the figure 2. And now, this configuration corresponds with HELIAS stellarator with five periods of the magnetic filed and consequently with five magnetic islands [7,8].

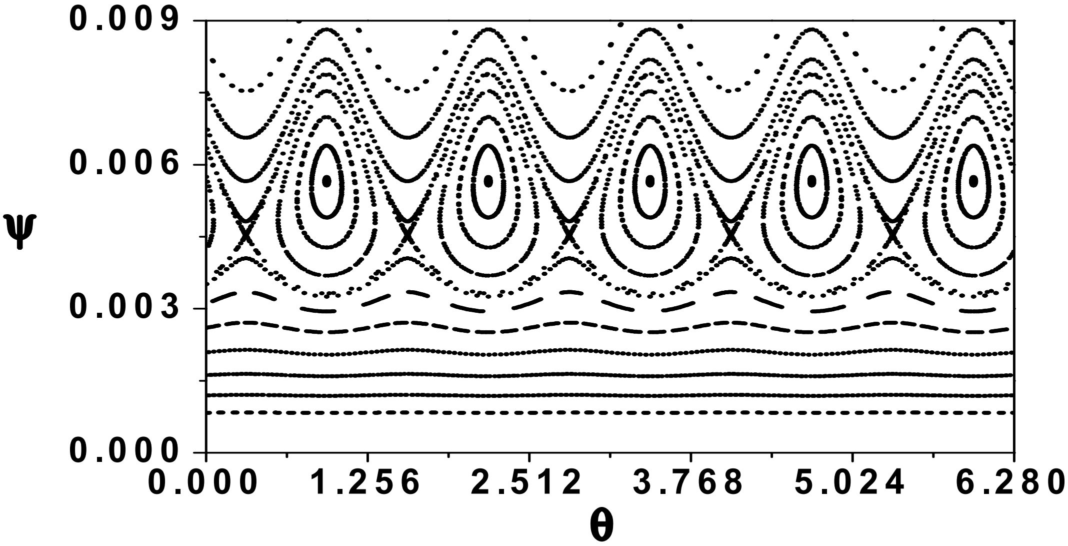

Fig.2. Vertical cross- section of the magnetic field configuration in the toroidal system with one island chain in the flux coordinates.

There is a number of papers devoted to the particle distribution function and transport calculations for the magnetic filed configurations similar to the one presented on figure 1 [9,10]. In this paper we are going to describe the changes of the guiding center distribution function, which can take place when we include magnetic field perturbation in our configuration. This task is of high importance for the modern fusion devices as far as the knowledge of the distribution function with the resonant structures gives us possibility to estimate the particle fluxes in the plasma edge regions, where the island structures are the dominant. In comparison with previous attempts to represent the effect of magnetic field space perturbations in toroidal plasmas [11] we propose to use the technique, which includes the redefinition of distribution function for the resonant magnetic field configuration. This technique will be presented further in more detail. As far as heavy impurities with high charge states have the strong tendency to effect on the background plasma and on the plasma transport in the edge regions, the knowledge of impurities distribution is of high importance as well. The numerical calculation is done for the tungsten ions, which are expected to be the dominant impurities in the HELIAS stellarator.

GUIDING CENTER EQUATIONS IN FLUX COORDINATES

The guiding center equations of Hamiltonian form are the following [4,5,6],

$$\dot{P}_\zeta = \rho_\parallel \frac{\partial \alpha}{\partial \zeta} B^2 - \left(\rho_\parallel^2 B + \mu\right)\frac{\partial B}{\partial \zeta} - \frac{\partial \Phi}{\partial \zeta}, \tag{1}$$

$$\dot{P}_\vartheta = \rho_\parallel \frac{\partial \alpha}{\partial \vartheta} B^2 - \left(\rho_\parallel^2 B + \mu\right)\frac{\partial B}{\partial \vartheta} - \frac{\partial \Phi}{\partial \vartheta}, \tag{2}$$

$$\dot{\zeta} = \left[\rho_\parallel B^2\left(1 + \rho_c \frac{\partial g}{\partial \psi} + \frac{\partial \alpha}{\partial \psi} I\right) - \frac{\partial \Phi}{\partial \psi} I - \left(\mu + \rho_\parallel^2 B\right)\frac{\partial B}{\partial \psi} I\right]\frac{1}{D}, \tag{3}$$

$$\dot{\vartheta} = \left[\rho_\parallel B^2\left(\iota - \rho_c \frac{\partial g}{\partial \psi} - \frac{\partial \alpha}{\partial \psi} g\right) + \frac{\partial \Phi}{\partial \psi} g + \left(\mu + \rho_\parallel^2 B\right)\frac{\partial B}{\partial \psi} g\right]\frac{1}{D}, \tag{4}$$

$$\dot{\psi} = \frac{g}{D}\dot{P}_\vartheta - \frac{I}{D}\dot{P}_\zeta, \tag{5}$$

$$\dot{\rho}_{\|} = \frac{\rho_{\|}^2 B + \mu}{D}\left\{\left[\rho_c \frac{\partial g}{\partial \psi} - \iota + g\frac{\partial \alpha}{\partial \psi}\right]\frac{\partial B}{\partial \vartheta} + \left[I\frac{\partial \alpha}{\partial \zeta} - g\frac{\partial \alpha}{\partial \vartheta}\right]\frac{\partial B}{\partial \psi} - \right.$$
$$\left. - \left[I\frac{\partial \alpha}{\partial \psi} + 1 + \rho_c\frac{\partial g}{\partial \psi}\right]\frac{\partial B}{\partial \zeta}\right\} - \frac{1}{D}\left[\left(\iota - \rho_c\frac{\partial g}{\partial \psi}\right)\frac{\partial \Phi}{\partial \vartheta} - \left(1 + \rho_c \frac{\partial g}{\partial \psi}\right)\frac{\partial \Phi}{\partial \zeta}\right] - \tag{6}$$
$$- \frac{\partial \alpha}{D\partial \psi}\left[I\frac{\partial \Phi}{\partial \zeta} - g\frac{\partial \Phi}{\partial \vartheta}\right] - \frac{\partial \Phi}{D\partial \psi}\left[g\frac{\partial \alpha}{\partial \vartheta} - I\frac{\partial \alpha}{\partial \zeta}\right] - \frac{\partial \alpha}{\partial t}.$$

Numerical integration of the equations (1)-(6) makes it possible to create not only the model of charged particle motion in the magnetic and electric fields but the models of the magnetic field lines as well, which have been presented above on figures 1 and 2. One of the advantages of these equations is the possibility to separate terms, which are responsible for taking into account the magnetic field perturbations. For example, if we have the magnetic configuration with closed nested magnetic surfaces without any perturbations like that shown on figure 1, the guiding center equations that we need to present such case are as follows,

$$\dot{P}_{\vartheta 0} = -\left(\rho_{\|}^2 B + \mu\right)\frac{\partial B}{\partial \vartheta} - \frac{\partial \Phi_0}{\partial \vartheta}, \tag{7}$$

$$\dot{P}_{\zeta 0} = -\left(\rho_{\|}^2 B + \mu\right)\frac{\partial B}{\partial \zeta} - \frac{\partial \Phi_0}{\partial \zeta}, \tag{8}$$

$$\dot{\zeta}_0 = \left[\rho_{\|} B^2\left(1 + \rho_c\frac{\partial g}{\partial \psi}\right) - \frac{\partial \Phi_0}{\partial \psi}I - \left(\mu + \rho_{\|}^2 B\right)\frac{\partial B}{\partial \psi}I\right]\frac{1}{D}, \tag{9}$$

$$\dot{\vartheta}_0 = \left[\rho_{\|} B^2\left(\iota - \rho_c\frac{\partial g}{\partial \psi}\right) + \frac{\partial \Phi_0}{\partial \psi}g + \left(\mu + \rho_{\|}^2 B\right)\frac{\partial B}{\partial \psi}g\right]\frac{1}{D}, \tag{10}$$

$$\dot{\psi}_0 = \frac{g}{D}\dot{P}_{\vartheta 0} - \frac{I}{D}\dot{P}_{\zeta 0}, \tag{11}$$

$$\dot{\rho}_{\|0} = \frac{\rho_{\|}^2 B + \mu}{D}\left\{\left[\rho_c\frac{\partial g}{\partial \psi} - \iota\right]\frac{\partial B}{\partial \vartheta} - \left[1 + \rho_c\frac{\partial g}{\partial \psi}\right]\frac{\partial B}{\partial \zeta}\right\} -$$
$$- \frac{1}{D}\left[\left(\iota - \rho_c\frac{\partial g}{\partial \psi}\right)\frac{\partial \Phi_0}{\partial \vartheta} - \left(1 + \rho_c\frac{\partial g}{\partial \psi}\right)\frac{\partial \Phi_0}{\partial \zeta}\right]. \tag{12}$$

If we want to present the magnetic field perturbation and describe the configuration presented on figure 2, we have to add the following terms,

$$\dot{P}_{\zeta 1} = \rho_{\|}\frac{\partial \alpha_1}{\partial \zeta}B^2 - \frac{\partial \Phi_1}{\partial \zeta}, \tag{13}$$

$$\dot{P}_{\vartheta 1} = \rho_{\|}\frac{\partial \alpha_1}{\partial \vartheta}B^2 - \frac{\partial \Phi_1}{\partial \vartheta}, \tag{14}$$

$$\dot{\zeta}_1 = \left(\rho_{\|} B^2\frac{\partial \alpha_1}{\partial \psi} - \frac{\partial \Phi_1}{\partial \psi}\right)\frac{I}{D}, \tag{15}$$

$$\dot{\vartheta}_1 = \left(-\rho_{\|} B^2\frac{\partial \alpha_1}{\partial \psi} + \frac{\partial \Phi_1}{\partial \psi}\right)\frac{g}{D}, \tag{16}$$

$$\dot{\psi}_1 = \frac{g}{D}\dot{P}_{\vartheta 1} - \frac{I}{D}\dot{P}_{\zeta 1}, \tag{17}$$

$$\dot{\rho}_{\|1} = \frac{\rho_\|^2 B + \mu}{D}\left\{ g\frac{\partial\alpha_1}{\partial\psi}\frac{\partial B}{\partial\vartheta} + \left[I\frac{\partial\alpha_1}{\partial\zeta} - g\frac{\partial\alpha_1}{\partial\vartheta}\right]\frac{\partial B}{\partial\psi} - \right.$$
$$\left. - I\frac{\partial\alpha_1}{\partial\psi}\frac{\partial B}{\partial\zeta}\right\} - \frac{1}{D}\left[\left(\iota - \rho_c\frac{\partial g}{\partial\psi}\right)\frac{\partial\Phi_1}{\partial\vartheta} - \left(1 + \rho_c\frac{\partial g}{\partial\psi}\right)\frac{\partial\Phi_1}{\partial\zeta}\right] - \tag{18}$$
$$- \frac{\partial\alpha_1}{D\partial\psi}\left[I\frac{\partial(\Phi_0+\Phi_1)}{\partial\zeta} - g\frac{\partial(\Phi_0+\Phi_1)}{\partial\vartheta}\right] - \frac{\partial(\Phi_0+\Phi_1)}{D\partial\psi}\left[g\frac{\partial\alpha_1}{\partial\vartheta} - I\frac{\partial\alpha_1}{\partial\zeta}\right] - \frac{\partial\alpha_1}{\partial t}.$$

Here the perturbation function α_1 is taken into account. This property of the guiding center equations is used in further analytical treatment of the problem.

DRIFT KINETIC EQUATION AND GUIDING CENTER DISTRIBUTION FUNCTION

The guiding center distribution function for the charged particles is considered as a function of three coordinate variables, longitudinal velocity, poloidal and toroidal momentums and time

$$f^* = f^*(\psi, \vartheta, \zeta, \rho_\|, P_\vartheta, P_\zeta, \tau). \tag{19}$$

As far as we consider the guiding center distribution function the kinetic equation for the particle distribution function transfers in to the drift kinetic equation in the following form

$$\frac{\partial f^*}{\partial\tau} + \frac{\partial f^*}{\partial\psi}\dot{\psi} + \frac{\partial f^*}{\partial\vartheta}\dot{\vartheta} + \frac{\partial f^*}{\partial\zeta}\dot{\zeta} + \frac{\partial f^*}{\partial P_\vartheta}\dot{P}_\vartheta + \frac{\partial f^*}{\partial P_\zeta}\dot{P}_\zeta = 0. \tag{20}$$

Here dotted variables present guiding center equations in the flux coordinates in general form (1-6). As far as we want to describe the changes of the guiding center distribution function, which can take place when we include the magnetic field perturbation, we present the solution for the equation (20) in the form of two terms

$$f^* = f_0^* + f_1^*. \tag{21}$$

Here the function f_0^* corresponds with the case of closed nested magnetic surfaces in toroidal configuration without any perturbation, and the function f_1^* will be considered in more detail further.

For the simplest case we consider the distribution function f_0^* in the Maxwell form

$$f_0^* = F(\psi)\exp\left[-\frac{\varepsilon}{T(\psi)}\right], \tag{22}$$

with the function $F(\psi) = n(\psi)T^{-\frac{3}{2}}(\psi)$ and the particle energy $\varepsilon = \frac{1}{2}\rho_\|^2 B^2 + \mu B + \Phi$ [4,6]. The radial dependences for the density and temperature are the following

$$n(r_p) = n(0)\left[1 + \left(\frac{r_p}{r_n}\right)^{2\alpha_n}\right]^{-1}, \tag{23}$$

$$T(r_p) = T(0)\left[1 + \left(\frac{r_p}{r_T}\right)^{2\alpha_T}\right]^{-1}. \tag{24}$$

Here the variable $r_p = \sqrt{2\psi}\frac{R_0}{a_p}$ is the normalized plasma radius, r_n, r_T and α_n, α_T are the parameters that define the steepness of the profiles, parameters R_0 and a_p are the large tore radius and plasma averaged radius respectively. Correspondent density and temperature profiles which are considered as typical for the HELIAS stellarator [8] are presented on figures 3 and 4.

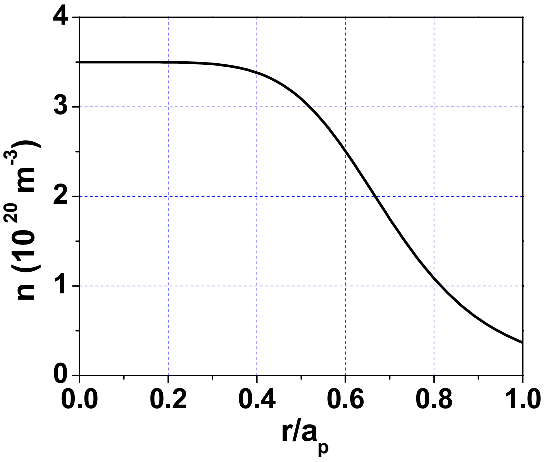

Fig.3. Typical density profile for the HELIAS stellarator with five periods of the magnetic field.

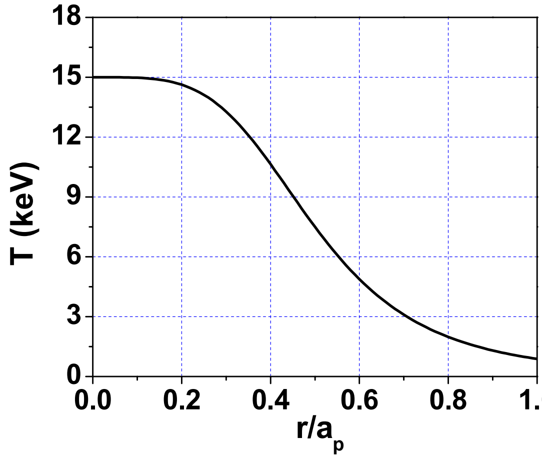

Fig.4. Typical temperature profile for the HELIAS stellarator with five periods of magnetic field.

The following figure 5 demonstrates the Maxwell distribution function of the form (22) in the velocity space $(\mu, \rho_{\parallel})$ with the use of the profiles (23, 24).

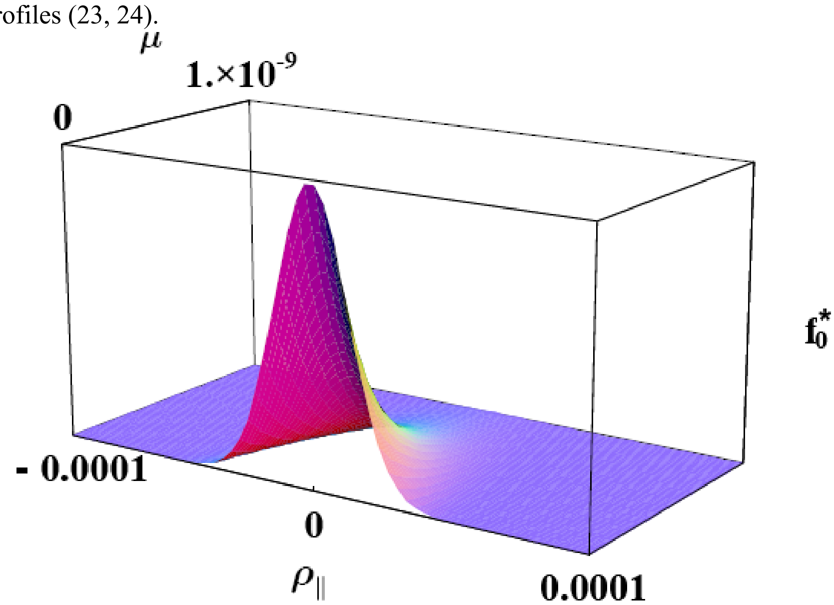

Fig.5. Maxwell guiding center distribution function in velocity space $(\mu, \rho_{\parallel})$.

Now let us proceed further and consider the second term f_1^* of the guiding center distribution function (21). This additional term is expected to appear when we take into account one magnetic field perturbation [4, 5]. In the vertical cross-section of the magnetic field configuration we will observe the island chain and as a result we have to expect changes in density and temperature profiles [12, 13]. It is obvious that such changes can be presented through the new distribution function with the help of term f_1^*. To get the solution for the equation (20) in the form (21), we just substitute (21) into (20) and take into account the possibility to separate the terms in the guiding center equations and the fact that the equilibrium distribution function f_0^* has to satisfy the following equation

$$\dot{\psi}_0 \frac{\partial}{\partial \psi} f_0^* + \dot{\vartheta}_0 \frac{\partial}{\partial \vartheta} f_0^* + \dot{\zeta}_0 \frac{\partial}{\partial \zeta} f_0^* + \dot{\rho}_{\parallel 0} \frac{\partial}{\partial \rho_{\parallel}} f_0^* + \dot{P}_{\vartheta 0} \frac{\partial}{\partial P_{\vartheta}} f_0^* + \dot{P}_{\zeta 0} \frac{\partial}{\partial P_{\zeta}} f_0^* = 0 . \tag{25}$$

As a result we obtain the drift kinetic equation in the linear approximation

$$\begin{gathered} \dot{\psi}_0 \frac{\partial}{\partial \psi} f_1^* + \dot{\psi}_1 \frac{\partial}{\partial \psi} f_0^* + \dot{\vartheta}_0 \frac{\partial}{\partial \vartheta} f_1^* + \dot{\vartheta}_1 \frac{\partial}{\partial \vartheta} f_0^* + \dot{\zeta}_0 \frac{\partial}{\partial \zeta} f_1^* + \\ + \dot{\rho}_{\parallel 0} \frac{\partial}{\partial \rho_{\parallel}} f_1^* + \dot{\rho}_{\parallel 1} \frac{\partial}{\partial \rho_{\parallel}} f_0^* + \dot{P}_{\vartheta 0} \frac{\partial}{\partial P_{\vartheta}} f_1^* + \dot{P}_{\zeta 0} \frac{\partial}{\partial P_{\zeta}} f_1^* = 0 , \end{gathered} \tag{26}$$

where dotted variables present the guiding center equations in the form (7-12) and in the form (13-18). Let us look for the solution f_1^* for the drift kinetic equation (26) in the form

$$f_1^* = f_{1\cos}^* \cos\chi_1 + f_{1\sin}^* \sin\chi_1, \tag{27}$$

where $f_{1\cos}^* = f_{1\cos}^*(\psi, \rho_\|, P_\vartheta, P_\zeta)$ as well as $f_{1\sin}^* = f_{1\sin}^*(\psi, \rho_\|, P_\vartheta, P_\zeta)$ are the functions of radial coordinate, longitudinal velocity and canonical momentums. χ_1 represents toroidal and poloidal dependencies as well as time dependence and initial phase dependence for the distribution function and reads $\chi_1 = n_1\zeta - m_1\vartheta - \omega_1\tau + \delta_1$. Now let us take into account some assumptions, which do not contradict with our model, but nevertheless simplify the drift kinetic equation. The electric field potential Φ_0 which corresponds with the magnetic field model presented on figure 1 has to be the function of flux surface and hence $\frac{\partial\Phi_0}{\partial\psi} \neq 0$, $\frac{\partial\Phi_0}{\partial\vartheta} = \frac{\partial\Phi_0}{\partial\zeta} = 0$. Also we assume that the unperturbed magnetic field has the uniform structure in the poloidal and toroidal directions $B \neq B(\vartheta,\zeta)$. That actually gives us possibility to represent the solution (27) trough the only $\sin\chi_1$ and $\cos\chi_1$ harmonics and take away the term $\dot{\vartheta}_1 \frac{\partial}{\partial\vartheta} f_0^*$ from the equation (26). Now we substitute the expression (27) into the equation (26), and with the assumptions mentioned above we get the drift kinetic equation of the form

$$\begin{gathered}\left(\rho_\|\iota + \frac{\partial\Phi}{\partial\psi}\right)\left(f_{1\cos}^* m_1 \sin\chi_1 - f_{1\sin}^* m_1 \cos\chi_1\right) + \rho_\|\left(f_{1\sin}^* n_1 \cos\chi_1 - f_{1\cos}^* n_1 \sin\chi_1\right) - \\ -\left(\rho_\| A_1 r_p^{m_1} m_1 \cos\chi_1 + \Phi_{\cos}^1 m_1 \sin\chi_1 - \Phi_{\sin}^1 m_1 \cos\chi_1\right)\frac{\partial f_0^*}{\partial\psi} + \\ + \left\{\begin{aligned} &-\left[\iota\left(\Phi_{\cos}^1 m_1 \sin\chi_1 - \Phi_{\sin}^1 m_1 \cos\chi_1\right) + \Phi_{\cos}^1 n_1 \sin\chi_1 - \Phi_{\sin}^1 n_1 \cos\chi_1\right] + \\ &+ A_1 r_p^{m_1} m_1 \frac{\partial\Phi_0}{\partial\psi}\cos\chi_1 + A_1 r_p^{m_1}\omega_1 \cos\chi_1 \end{aligned}\right\}\frac{\partial f_0^*}{\partial\rho_\|} = 0.\end{gathered} \tag{28}$$

In addition we have to outline that in the equation (28) the possible expected change of the electric field potential caused by changes of the guiding center distribution function is presented through the $\sin\chi_1$ and $\cos\chi_1$ harmonics as well and reads $\Phi_1 = \Phi_{\cos}^1 \cos\chi_1 + \Phi_{\sin}^1 \sin\chi_1$. This is the attempt to increase the level of self consistency of the model. Otherwise the terms that include Φ_1 can be simply dropped. If we separate terms with different harmonics we obtain two linear equations, which give us the expressions for the functions $f_{1\cos}^*$ and $f_{1\sin}^*$ as follows

$$f_{1\cos}^* = \frac{\Phi_{\cos}^1 m_1 \frac{\partial f_0^*}{\partial\psi} + \left(\iota m_1 + n_1\right)\Phi_{\cos}^1 \frac{\partial f_0^*}{\partial\rho_\|}}{\left(\rho_\|\iota + \frac{\partial\Phi_0}{\partial\psi}\right)m_1 - \rho_\| n_1}, \tag{29.a}$$

$$f_{1\sin}^* = \frac{\left(\rho_\| A_1 r_p^{m_1} m_1 - \Phi_{\sin}^1 m_1\right)\frac{\partial f_0^*}{\partial\psi} - \left(\iota\Phi_{\sin}^1 m_1 + \Phi_{\sin}^1 n_1 + A_1 r_p^{m_1}\left(m_1\frac{\partial\Phi_0}{\partial\psi} + \omega_1\right)\right)\frac{\partial f_0^*}{\partial\rho_\|}}{\left(\rho_\|\iota + \frac{\partial\Phi_0}{\partial\psi}\right)m_1 - \rho_\| n_1}. \tag{29.b}$$

The effect of the magnetic field perturbation is already observed through the resonant denominator. Analytical treatment of the 3D electric field profile at the edge plasma region, where the magnetic field configuration with the islands and stochastic layers is the dominant, is an extremely difficult task. Nevertheless we still can obtain some physics issues from the expressions (29.a) and (29.b). Let us completely neglect the electric field potential $\Phi_0 = \Phi_1 = 0$ and put $\rho_\| = const$, then the expression (29.a) simply reads as $f_{1\cos}^* = 0$ and the expression (29.b) becomes as follows

$$f_{1\sin}^{*}=A_1 r_p^{m_1} m_1 \frac{\partial f_0^*}{\partial \psi}\frac{1}{n_1-\iota m_1}. \tag{30}$$

When we put $\rho_{\parallel}=const$ we only restrict our consideration on the particles with the equal longitudinal velocity and further we can use the $\rho_{\parallel}$ as a parameter instead to use it as a variable. In such case as it is possible to see it is quite enough to use only $\sin\chi_1$ harmonic in distribution function representation (27). To resolve the problem of resonant denominator $n_1-\iota m_1$ one can use the technique described by Solov'ev and Shafranov [14]. Let us redefine the initial distribution function

$$f_0^* \to \int (n_1-\iota m_1)\frac{\partial f_0^*}{\partial \psi}d\psi . \tag{31}$$

Then the expression (30) reads as follows

$$f_{1\sin}^{*}=A_1 r_p^{m_1} m_1 \frac{\partial f_0^*}{\partial \psi}, \tag{32}$$

where f_0^* is the distribution function of the form (22). Now we can present the harmonic perturbation f_1^*

$$f_1^{*}=A_1 r_p^{m_1} m_1 \frac{\partial f_0^*}{\partial \psi}\sin\chi_1 . \tag{33}$$

Keeping in mind all the assumptions mentioned above, we obtain the solution for the drift kinetic equation (20) in the form

$$\begin{aligned} f^* = \int (n_1-m_1\iota)\exp\left(-\frac{\varepsilon}{T(\psi)}\right)\left(\frac{\partial F(\psi)}{\partial \psi}+F(\psi)\frac{\varepsilon}{T^2(\psi)}\frac{\partial T(\psi)}{\partial \psi}\right)d\psi + \\ + A_1 r_p^{m_1} m_1 \exp\left(-\frac{\varepsilon}{T(\psi)}\right)\left(\frac{\partial F(\psi)}{\partial \psi}+F(\psi)\frac{\varepsilon}{T^2(\psi)}\frac{\partial T(\psi)}{\partial \psi}\right)\sin\chi_1 . \end{aligned} \tag{34}$$

On figures 6a and 6b (different viewpoints) the guiding center distribution function of the form (34) in velocity space $(\mu,\rho_{\parallel})$ is presented. Space coordinates for the function f^* are chosen to calculate the distribution function in the region where the magnetic field separatrix occurs.

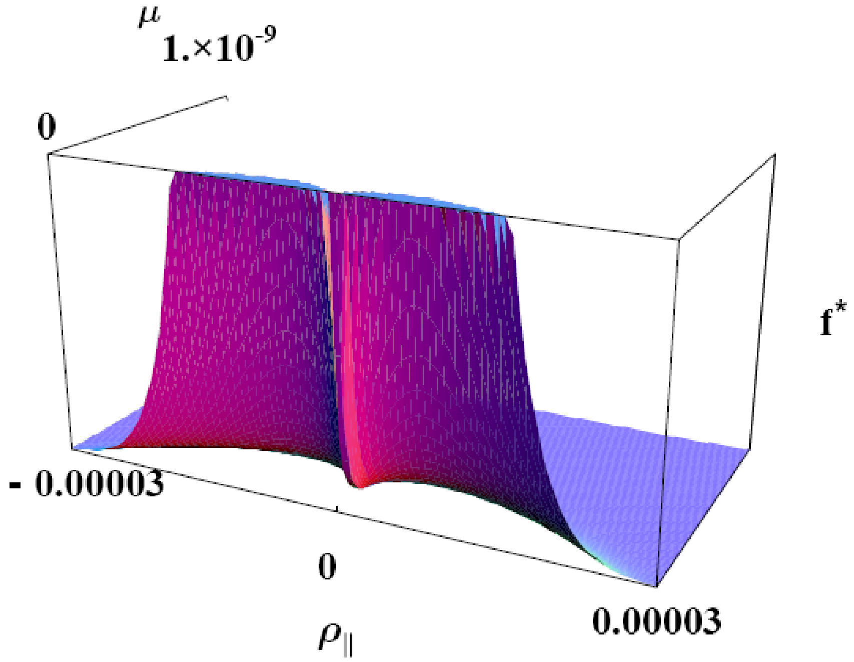

Fig. 6a. Guiding center distribution function of the form (34) in velocity space $(\mu,\rho_{\parallel})$.

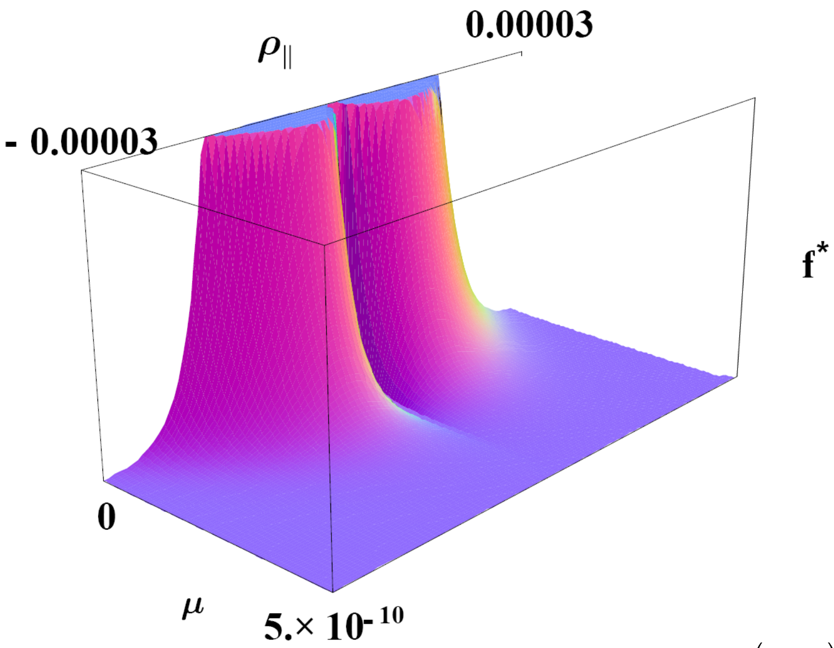

Fig. 6b. Guiding center distribution function of the form (34) in velocity space $(\mu, \rho_{\|})$.

Now let us consider the dependence of the guiding center distribution function on spatial coordinates for the charged particles with the constant energy. The tungsten ion is taken as a test particle with the energy $E = 1keV$, the ratio of the longitudinal velocity to the total velocity $v_k = 0.1$ and charge number $Z = 30$. On figure 7 the spatial dependence of initial guiding center distribution function (22) is presented. The longitudinal variable $\zeta = const$, poloidal angle variable ϑ varies $0 \le \vartheta \le 6.28$ and radial coordinate ψ varies $0.0034 < \psi < 0.0075$. We expect to observe the islands in this space region when we include the magnetic field perturbation.

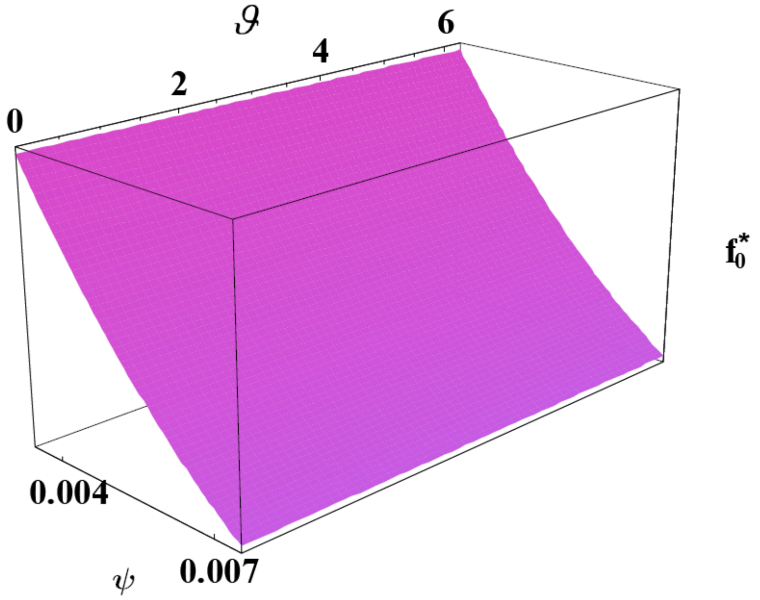

Fig. 7. Spatial topology of guiding center distribution function f_0^*.

The effect of the perturbation on the guiding center distribution function topology in space coordinates is presented on figure 8. As one can see five hills are formed in accordance with the island chain structure, which is presented on figure 2. Every separate island can be considered as the separate confinement system with the closed nested magnetic surfaces excluding the separatrix. Every local magnetic surface for the itch island now can be considered as a radial coordinate (in the local island coordinate system) to represent the guiding center distribution function within the separate island. This issue is typical for the island structures in the toroidal magnetic field configurations with open boundaries, when neither divertor nor limiter plates intersect the edge island configuration.

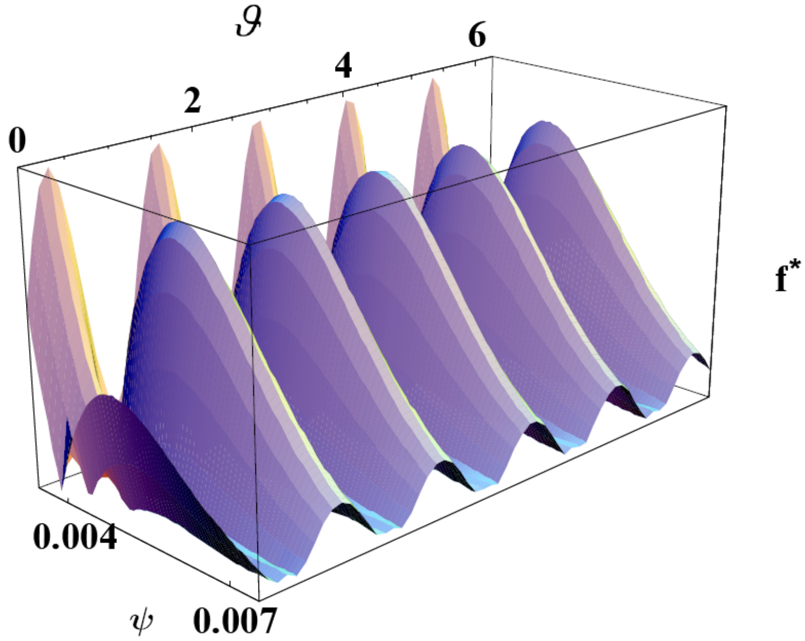

Fig. 8. Spatial topology of guiding center distribution function f^* of the form (34).

CONCLUSIONS

The analytical expression for the guiding center distribution function in the toroidal magnetic field configuration of the HELIAS stellarator with the five periods of the magnetic field and consequently with the five islands is derived. The general form of the new distribution function (34) does not have any limitation on particle spices and can be used to present either background particles distribution or impurity ions distribution as well. This is the critical task for the modern fusion devices as far as the knowledge of the distribution function of the form (34) gives us possibility to estimate the particle fluxes in the plasma edge regions where the magnetic islands are the dominant magnetic structures. Deriving the new distribution function we face with the problem of the denominator with zero values that correspond to the X-points and O-pints of the resonant magnetic structures (see the equation (30)). To avoid this problem we use the technique, which includes the redefinition of the initial distribution function, instead to include the collision term like it was done by previous authors [11]. This is the new main idea, to apply such approach to the consideration of the guiding center distribution function.

We consider the case, in which the electric field potential is neglected. At the same time we keep in mind the idea to obtain the electric field profile for the island region using the new background plasma profiles and the new impurity ion profiles derived with the use of the distribution function (34). It is shown that in the drift kinetic equation solution (27) the $\cos\chi_1$ harmonic vanishes due to the mentioned above approximation and only the one $\sin\chi_1$ harmonic represents changes of the guiding center distribution function caused by the magnetic field perturbation.

Analytical treatment of the problem is undertaken in two steps. The first step is to consider the magnetic filed without any perturbation (magnetic field configuration is shown on figure 1) and to present the initial guiding center distribution function with corresponding temperature and density profiles. The second step is to include magnetic field perturbation (figure 2) and to derive the changes in distribution function.

As far as the impurities have a strong effect on the plasma performance and the tungsten is expected to be the dominant impurity in the HELIAS stellarator plasma, for the numerical test the tungsten ion distribution function is considered. The initial guiding center distribution function is taken in Maxwell form (22). The spatial topology of the guiding center distribution function calculated from the formula (34) for the ions with fixed energy is presented on figure 8. Five "hills" corresponding with the five main magnetic islands of HELIAS stellartator are observed.

The presentation of the guiding center distribution function in velocity space $(\mu, \rho_{\|})$ has to be studied in more details. This will be done in future papers.

ACKNOWLEDGMENT

Authors deeply thank Prof. Dr. Alexander Shishkin and Dr. Igor Girka for numerous and fruitful discussions.
This work is carried out under the Science and Technology Center in Ukraine Project No 2313.

REFERENCES

1. E.D. Volkov, V.A. Suprunenko, A.A. Shishkin, "STELLARATOR". - Kiev, Naukova Dumka, 1983, /in Russian/.
2. A.I. Morozov and L.S. Solov'ev 1966 *Motion of Charged Particles in Electromagnetic Fields* // (Reviews of Plasma Physics vol 2) ed M.A. Leontovich (New York: Consultant Bureau) P. 201-297.
3. W.D. D'haeseller, W.N.G. Hitchon, J.D. Callen, J.L. Shoet, *Flux Coordinates and Magnetic Field Structure* // Springer-Verlag Berlin Heidelberg 1991.
4. R.B. White and M.S. Chance, *Hamiltonian Guiding Center Drift Orbit Calculation for Plasmas of Arbitrary Cross Section* // Phys. Fluids, October 1984. -Vol. 27. -N 10. -P. 2455-2467.
5. O.A. Shishkin, *Resonant Islands in the Magnetic Traps of Stellarator Type* // "Visnyk KNU", 2000. -N 496. issue 4/12/. -P. 34-38 /in Russian/.
6. Nickolai A. Azarenkov, Oleg A. Shyshkin, Ralf Schneider, Horst Wobig, Yurii L. Igitkhanov, *Stochasticity of heavy ion trajectories in drift optimised plasma configuration* // "Visnyk KNU", 2002. –N 574, issue 4/20/. –P.27-43.
7. T. Amano, C.D.Beidler, E. Harmeyer, F. Herrnegger, A.Kendl, J. Kisslinger, C. Nuerenberg, I. Sidorenko, E. Strumberger, H. Wobig, *Progress in Helias Reactor Studies* // 12th Int. Stellarator Workshop, Madison, Wisconsin, Sept.29–Oct.1, 1999.
8. C.D. Beidler, *Neoclassical Transport Properties of HSR* // Proceedings of the 6th Workshop on Wendelstein 7-X and Helias Reactor, January (1996), IPP 2/331. –P. 194-201.
9. Allen H. Boozer and Gioietta Kuo-Petravic, *Monte Carlo evaluation of transport coefficients* // Phys. Fluids, May 1981. –Vol. 24. –N 5. –P. 851-859.
10. C.D. Beidler and H. Maassberg, *An improved formulation of the ripple-averaged kinetic theory of the neoclassical transport in stellarators* // Plasma Phys. Control. Fusion, August 2001. –Vol. 43. –N 8. –P. 1131-1148.
11. G. Casati, E. Lazzaro and A. Orefice, *Effect of magnetic field space perturbations in toroidal plasmas in the intermediate diffusion regime* // Phys. Fluids, April 1974. –Vol. 17. –N 4, -P. 847-848.
12. Y. Nakamura, Y. Takeiri, R. Kumazawa, M. Osakabe, T. Seki et al, *Plasma performance and impurity behavior in long pulse discharges on LHD* // Nuclear Fusion. –2003. -Vol.43. –P. 219-227.
13. K. Ida, S. Inagaki, N.Tamura et al, *Radial electric field and transport near the rational surface and the magnetic island in LHD* // Nuclear Fusion. –2004. –Vol.44. –P. 290-295.
14. L.S. Solov'ev and V.D. Shafranov 1970 *Closed Magnetic Configurations for Plasma Confinement* // (Reviews of Plasma Physics vol 5) ed M.A. Leontovich (New York: Consultants) –P. 1-247.

ФУКЦИЯ РАСПРЕДЕЛЕНИЯ ВЕДУЩИХ ЦЕНТРОВ В ТОРОИДАЛЬНОМ МАГНИТНОМ ПОЛЕ С ОДНОЙ РЕЗОНАНСНОЙ СТРУКТУРОЙ

Николай А. Азаренков*, Олег А. Шишкин*,
Ральф Шнайдер, Юрий Л. Игитханов****
**Харьковский национальный университет им.В.Н.Каразина, пл.Свободи 4, 61077 Харьков, Украина*
***Институт физики плазмы имени Макса Планка, ассоциация ЕВРАТОМ, отделение Грайфсвальд, D-17489, Грайфсвальд, Германия*

В данной работе, путем решения дрейфового кинетического уравнения, получено аналитическое выражение для функции распределения ведущих центров заряженных частиц в тороидальном магнитном поле стелларатора HELIAS с одной цепочкой магнитных островов (одной резонансной структурой). Используя полученную функцию распределения мы можем построить выражения для потоков частиц в трехмерной магнитной конфигурации с учетом островных структур, что является одной из основных задач теории современного термоядерного синтеза. Дополнительные возмущения, приводящие к появлению стохастических слоев, не учитываются. Также в модели не учитывается наличие электрического поля. Решение дрейфового кинетического уравнения ищется в виде суммы гармоник $\sin\chi_1$ и $\cos\chi_1$, где χ_1 описывает зависимость искомой функции от полоидальной и тороидальной угловых переменных, "волновых" чисел возмущения, частоты возмущения и времени. В работе показано, что при учете выше упомянутых предположений решение дрейфового кинетического уравнения сводится к виду содержащему зависимость от угловых переменных только в виде гармоники $\sin\chi_1$. Этого вполне достаточно для описания изменений функции распределения, вызванных появлением цепочки островов магнитного поля. Численные примеры приведены для функции распределения ведущих центров ионов вольфрама.

КЛЮЧЕВЫЕ СЛОВА: HELIAS, стелларатор, уравнения ведущего центра, функция распределения ведущих центров, резонанс магнитного поля, потенциал электрического поля