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FINE STRUCTURE OF ANGULAR DISTRIBUTIONS AND POLARIZATION OF RADIATION BY RELATIVISTIC ELECTRONS AND POSITRONS IN A THIN CRYSTAL

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The results of the theoretical investigation of angular distributions and polarization of radiation by relativistic electrons and positrons passing through a thin crystal at a small angle to the crystal axis are presented. It is shown that the nontrivial fine structure of angular distributions of the emitted photons is connected to the multiple scattering effect on radiation of high energy particles in crystal (similar to the Landau-Pomeranchuk-Migdal effect). It is also shown that using slit collimation of photon beam one can obtain a high degree of linear polarization of radiation, however, the circular polarization of radiation in a thin crystal is close to zero for any direction.

KEY WORDS: relativistic electron, relativistic positron, gamma radiation, non-dipole regime, linear polarization, circular polarization, crystal.

The recent progress in creation of high energy accelerators requires recheck and more accurate definition of some predictions of the theory of high energy particle interaction with matter. First of all it is a question of electrodynamics effects that appears at ultra high energies. One of them is the Landau-Pomeranchuk-Migdal effect (LPM effect) of suppression of radiation of relativistic particles in an amorphous matter, which was predicted half a century ago by Landau and Pomeranchuk [1]. The quantitative theory of the effect in an amorphous medium was offered by Migdal using the kinetic equation method [2]. However, the experimental verification of the theory was carried out only several years ago at SLAC for electron energies 8 and 25 GeV [3].

It should be noted that there were some previous attempts of experimental check of the LPM effect in cosmic rays and accelerator, but there was lack of statistic at these experiments for quantitative verification. The topicality of such investigation today is dictated by the fact that due to the LPM effect the radiation length at ultra high energy can increase considerably. This leads to the increasing of effective length of electromagnetic cascade in matter that necessary to take into account at designing detectors and radiation protection for a new generation of high energy accelerators. The increasing of radiation length due to the LPM effect was observed recently at CERN at electron energies up to 300 GeV [4].

It is significant that the experimental study of the LPM effect carried out at SLAC showed a good agreement of measured data with the Migdal's theory predictions for rather thick targets [3]. However, in the case of thin (in comparison with coherence length) targets there was observed the essential discrepancy between theory and experiment [3,5]. The reason of the discrepancy lays in the fact that Migdal developed the theory of LPM effect for boundless amorphous media. Taking into account a finite thickness of target leads to considerable changes of radiation spectrum of relativistic electrons that was predicted earlier in Ref. [6-8]. Thus, the SLAC experiment confirmed the main results of Migdal's theory for thick enough targets and stimulated the further theoretical investigations with the aim to develop the quantitative theory of the suppression effect for radiation in a thin layer of matter. Such a theory was developed in works [9-13].

Recently it was shown [14] that multiple scattering of relativistic electrons by atoms in amorphous medium could influence not only radiation spectrum but also the angular distribution of emitted gamma-quanta. Much more significant changes of spectral and angular distribution of radiation takes place in a crystal [8,15] due to coherent effects both at scattering and radiation of relativistic electrons.

In present work the results of theoretical study of the multiple scattering effect on spectral-angular distribution and polarization characteristics of radiation by ultra relativistic electrons and positrons in a thin crystal are presented. The calculations are carried out for 200 GeV electrons (positrons) impinging the 20 μm tungsten crystal under the small angle to the axis <111>. Such a conditions corresponds to the conditions of recent CERN experiment [16]. The results of this study show that the angular distributions of radiation of ultra relativistic electrons and positrons have a nontrivial thin structure, which is caused by the coherent effect in particle scattering by atomic rows of a crystal and essentially nondipole regime of radiation at so high energy. It is shown that rather high degree of linear polarization can be obtained in this case using slit-type photon collimator. It is also shown that the circular polarization of emitted gamma-quanta in a thin crystal is close to zero for any direction.

GENERAL FORMULAS

The radiation process of a relativistic electron develops in a large spatial region along the direction of particle motion, which is called the coherence length [17] $l_c \approx 2\gamma^2 \omega^{-1}$, where γ is the Lorenz-factor of the particle, ω is the energy of emitted photon (we use here the units system $\hbar = c = 1$). This length grows fast with electron energy increasing and with decreasing of photon energy ω.

Landau and Pomeranchuk showed [1] that if in limits of coherence length of radiation an electron interacts with a large number of medium atoms, the multiple scattering of particle on these atoms could conduce to suppression of bremsstrahlung in comparison with the prediction of the Bethe-Heitler theory [18].

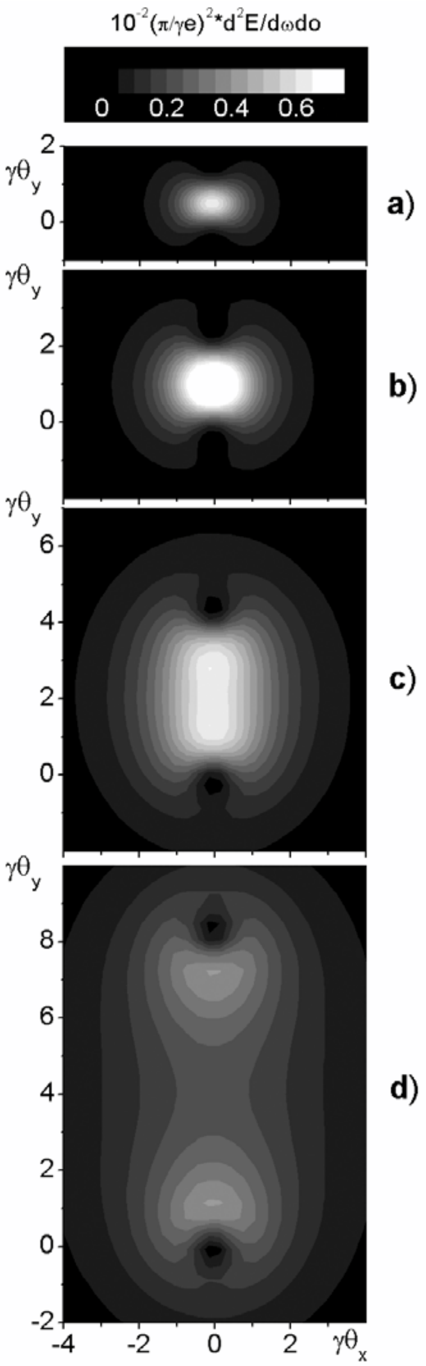

Fig.1. Angular distribution of radiation by one electron scattered on the angle $\gamma\vartheta_e$. a) $\gamma\vartheta_e = 1$; b) $\gamma\vartheta_e = 2$; c) $\gamma\vartheta_e = 4$; d) $\gamma\vartheta_e = 8$

The main condition of the LPM effect coincides with the condition of non-dipole regime of radiation

$$\vartheta_e > \gamma^{-1} \quad (1)$$

It can be fulfilled first of all for relatively low energy region of emitted photons $\omega \ll \varepsilon$ so as to provide a long enough coherence length l_c. In this case we can neglect the quantum recoil effect at radiation and use formulas of classical electrodynamics.

The spectral-angular density of radiation by an electron of trajectory $\vec{r}(t)$ is determined in classical electrodynamics by the expression [19]

$$\frac{d^2E}{d\omega do} = \frac{e^2}{4\pi^2}\left[\vec{k} \times \vec{I}\right]^2, \; \vec{I} = \int_{-\infty}^{\infty} \vec{v}(t)\, e^{i(\omega t - \vec{k}\vec{r})} dt, \quad (2)$$

where $\vec{k}$ and ω are the wave vector and the frequency of the radiated wave.

If the coherence length of radiation process is big in comparison with the thickness of the target $l_c \gg T$ then $\vec{I}$ can be represented as [7,19]

$$\vec{I} \approx \frac{i}{\omega}\left(\frac{\vec{v}'}{1 - \vec{n}\vec{v}'} - \frac{\vec{v}}{1 - \vec{n}\vec{v}}\right), \quad (3)$$

where $\vec{v}$ and $\vec{v}'$ are the electron velocities before and after scattering, $\vec{n} = \vec{k}/\omega$.

The two-dimensional picture of the angular distributions of radiation by a relativistic electron scattered at the different angles ϑ_e is given in Fig. 1. This picture shows a nontrivial angular distribution of essentially non-dipole electron radiation ($\gamma\vartheta_e \gg 1$) with sharp maxima and deep minima near the initial and final directions of the electron motion. The maximum value of radiation intensity is reached at $\gamma\vartheta_e = 2$ (see Fig. 1b).

The general expression for the polarization matrix of radiation is

$$J_{ik} = \frac{e^2\omega^2}{4\pi^2}\left(\vec{e}_i\vec{I}\right)\left(\vec{e}_k\vec{I}^*\right), \quad (4)$$

where $\vec{e}_{i,k}$ are the polarization vectors which are the unit vectors orthogonal to the wave vector $\vec{k}$ and to each other: $\vec{e}_{i,k}\vec{n} = 0$, $\vec{e}_i\vec{e}_k = \delta_{ik}$.

The degree of the linear polarization of radiation is determined by expression

$$P = \frac{J_{11} - J_{22}}{J_{11} + J_{22}}. \quad (5)$$

The degree of the circular polarization of radiation is

$$P_{circular} = \frac{J_{12} - J_{21}}{J_{11} + J_{22}}. \quad (6)$$

In [20] there was proposed a method for producing high energy gamma-quanta with circular polarization by coherent radiation of relativistic electrons passing through a crystal at a small angle to the crystal axis.

This method is based on specific features of non-dipole regime of radiation that allows obtaining a high enough degree (about 50 %) of circular polarization of emitted gamma-quanta using the special type of photon collimation. The natural limitation of efficiency of this method for hard part of the radiation spectrum is connected with rapid decreasing of the number of emitted gamma-quanta with photon energy increasing $dN/d\omega \sim 1/\omega$. However, there is one more effect that suppresses the emission of circular polarization photons in a soft part of the spectrum despite the increasing of the total number of emitted photons.

Really, if the energy of the emitted photon is small enough, so that the coherence length of radiation process is larger than crystal thickness ($l_c >> T$) then the polarization matrix (4) in this approximation becomes symmetrical

$$J_{ik} = J_{ki},$$

and the degree of circular polarization of each emitted gamma-quanta (6) is equal to zero.

This statement can be formulated as the following general theorem: if the coherence length of radiation process is much larger than the effective spatial region in which a relativistic electron interacts with an external field, then the circular polarization of the emitted photon is identically zero.

The spectral-angular density of radiation can be written in terms of polarization matrix as

$$\frac{d^2E}{d\omega do} = J_{11} + J_{22}. \tag{7}$$

If we are interested in the angular distribution of radiation from an electron beam passing through a thin target, then the formula (7) is necessary for averaging over the scattering angles of the particles in matter. If the distribution function of the scattered particles $f(\vec{\vartheta}_e)$ is known, then the average value of spectral-angular density of radiation will be determined by the expression

$$\left\langle \frac{d^2E}{d\omega do} \right\rangle = \int d\vec{\vartheta}_e \, f(\vec{\vartheta}_e) \frac{d^2E}{d\omega do}. \tag{8}$$

Note that formula (8) is applicable to any targets. It is only required that the target thickness is small in comparison with the coherence length of radiation. The different characteristics of the scatterer will be exhibited only by the definite kinds of distribution function $f(\vec{\vartheta}_e)$.

For the amorphous target case the distribution function $f(\vec{\vartheta}_e)$ is determined by the Bethe-Molière function [19]. The multiple scattering effect on the spectral-angular distribution of the radiation by relativistic electrons in a thin amorphous target was studied in [14]. It was shown that in contrast with the Bethe-Heitler theory prediction [18] there is the minimum in the angular distribution of the emitted gamma-quanta in the initial direction of the electron beam when the condition of the LPM effect $\vartheta_e >> 1/\gamma$ is fulfilled (see Fig. 2 in [14]).

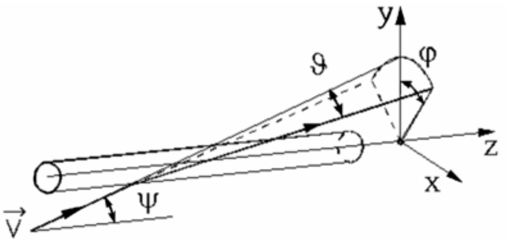

Fig.2. Geometry of the coherent azimuthal scattering of electron in the field of atomic string of crystal, $\vartheta = 2\psi \sin\varphi/2$

If a beam of relativistic electrons (positrons) passes through a crystal at a small angle ψ to one of crystallographic axes (axis z) there takes place a coherent effect in electron scattering, exhibited as a characteristic annular angular distributions of the particles outgoing from the crystal (“doughnut scattering” effect [19,21]). This coherent effect takes place only for the scattering over azimuthal angle φ (see Fig. 2) as a result of correlations in sequent scattering of a fast electron by atoms located along this crystal axis.

In this case the magnitude of the root-mean-square angle of multiple scattering of electron can exceed substantially (by several times) the corresponding parameter for the electron scattering in an amorphous target of the same thickness [21], and the smaller is the target thickness, the greater is this difference.

Generally the dynamics of a relativistic particle beam in an aligned crystal is rather complicated, since various fractions of a beam are involved in various regimes of motion: finite and infinite, regular and chaotic, with transitions between them. The analytical description of the particle dynamics can be conducted only in some limiting cases. Thus, for example, the theory of multiple scattering of relativistic charged particles on atomic strings of a crystal, based on the continuous string approximation, describes the coherent azimuthal scattering of above-barrier electrons [21].

However, this theory does not describe transitions of particles between two different fractions of the electron beam in the crystal, since the continuous string approximation does not take into account incoherent scattering. It is possible to take incoherent scattering into account by analytical methods only in the case of rather large incident angles $\psi >> \psi_L$, where $\psi_L = \sqrt{4Ze^2/\varepsilon d}$ is the Lindhard angle [19], Z|e| is the charge of atomic nuclear, *d* is the distance between atoms along the crystal axis and ε is the energy of the incident electron. At the same time, as it was already mentioned, the orientation effects in scattering and radiation of a relativistic electron beam passing through a crystal are mostly manifest in the range of angles $\psi \approx \psi_L$. Therefore, for the quantitative description of these effects, a computer simulation of the passing of an electron beam through an aligned crystal appears to be the most adequate.

RESULTS OF COMPUTER SIMULATION

With the purpose of a quantitative analysis of the multiple scattering effect on coherent radiation of a relativistic electron (positron) in a thin crystal, we performed a computer simulation on the basis of the Monte-Carlo method. We used here the binary collisions model of the electron interactions with the atoms of a crystalline lattice [15]. Such an approach allows taking into account both the coherent scattering of fast electrons on the atomic strings of a crystal and the incoherent scattering of the electrons connected with the thermal fluctuations of the atom positions in the lattice and with the electronic subsystem of the crystal. The angular distribution of radiation and polarization was calculated as a sum of radiation and polarization of each particle. The results of the computer simulation of electron (positron) scattering, radiation and polarization are presented in Fig. 3.

The left part of Fig. 3a represents the results of computer simulation of the angular distribution of 200 GeV electrons (positrons) scattered by 20 μm tungsten crystal at random orientation (like in amorphous target). The initial divergence of the electron beam was taken as 1 μrad that is equal to 0.4 in the units of $1/\gamma$. The root-mean-square angle of multiple scattering $<\vartheta_e>$ for this case is about $3/\gamma$. It means that we have non-dipole regime of radiation even at random orientation of 20 μm tungsten crystal with the "non-dipole" parameter is $\gamma <\vartheta_e> \approx 3$. The Fig. 3a (middle) shows the corresponding angular distribution of emitted gamma-quanta. In contrast to the dipole radiation case there are the deep minimum along the initial electron beam direction. We use here the units $1/\gamma$ that is a natural scale for angular distributions of relativistic particle radiation.

Fig. 3b (left) demonstrate the typical for the "doughnut scattering" effect annular angular distribution of scattered electrons when initial electron beam impinges on the crystal at the Lindhard angle $\psi = \psi_L$. The position of the crystal axis <111> is shown in Fig. 3b (left) by black cross. The initial direction of the electron beam is in the center of black square on this figure. The black square is also indicates the angular region where the gamma-radiation is mainly located. Namely this region is represented in Fig. 3b (middle) with corresponding zooming.

The results of analogous calculations for 200 GeV positron beam in the tungsten crystal of the same thickness and orientation are presented in Fig. 3c. One can see that angular distribution of scattered electrons (Fig. 3b, left) and positrons (Fig. 3c, left) is rather different. This difference is connected with the character of the motion of positrons and electrons in a lattice field. As a result we have also different structure of angular distribution of radiation and its polarization (see Figs. 3b and 3c, right).

The integral degree of linear polarization of electron radiation is about zero. However, using the slit-type horizontal photon collimator with the angular width $\Delta\theta = \gamma^{-1}$ and putting them as it sowed in Fig.3b (right) by dashed lines it is possible to obtain linearly polarized photon beam with polarization degree about 80 %.

Note, that just the special type of coherent scattering in a crystal and the non-dipole regime of radiation allow us to obtain a high degree linearly polarized photon beams on the basis the angular separation of emitted gamma-quanta.

If we turn this collimator to the vertical position (in parallel to y-axis) we obtain the linearly (vertically) polarized photon beam with about 80% of polarization degree. Note, that such a behavior of polarization is very similar to the amorphous target case but for the same efficiency of radiation it is necessary to use significantly thicker amorphous target.

Another situation for radiation of positron beam in crystal at the same condition. Horizontal collimation gives the result 75% of linear polarization, but the vertical one gives only 50 %.

All these features of angular distributions of radiation and polarization are associated with the superposition of two different effects: coherent electron scattering by the crystal rows ("doughnut scattering" effect) and suppression of radiation due to this coherent multiple scattering (analogous to the LPM effect in an amorphous target).

CONCLUSIONS

The present investigation shows a strong effect of the coherent multiple scattering of relativistic electrons in a thin crystal on the angular distributions of emitted gamma-quanta and their polarization.

It is shown that circular polarization of each photon emitted in a thin crystal ($T << l_c$) is close to zero.

The degree of linear polarization of collimated photon beam by the slit collimator with the angular width $\Delta\theta = \gamma^{-1}$ comes up to 80 % for 200 GeV electrons passed through the 20 μm tungsten crystal.

For the experimental observation of the effects described above a high angular resolution (better than γ^{-1}) of the gamma-detector is needed, as well as a small (less than γ^{-1}) divergence of the electron beam.

These effects must be taken into account when studying spectral-angular distributions of radiation and polarization by relativistic electrons in a crystal.

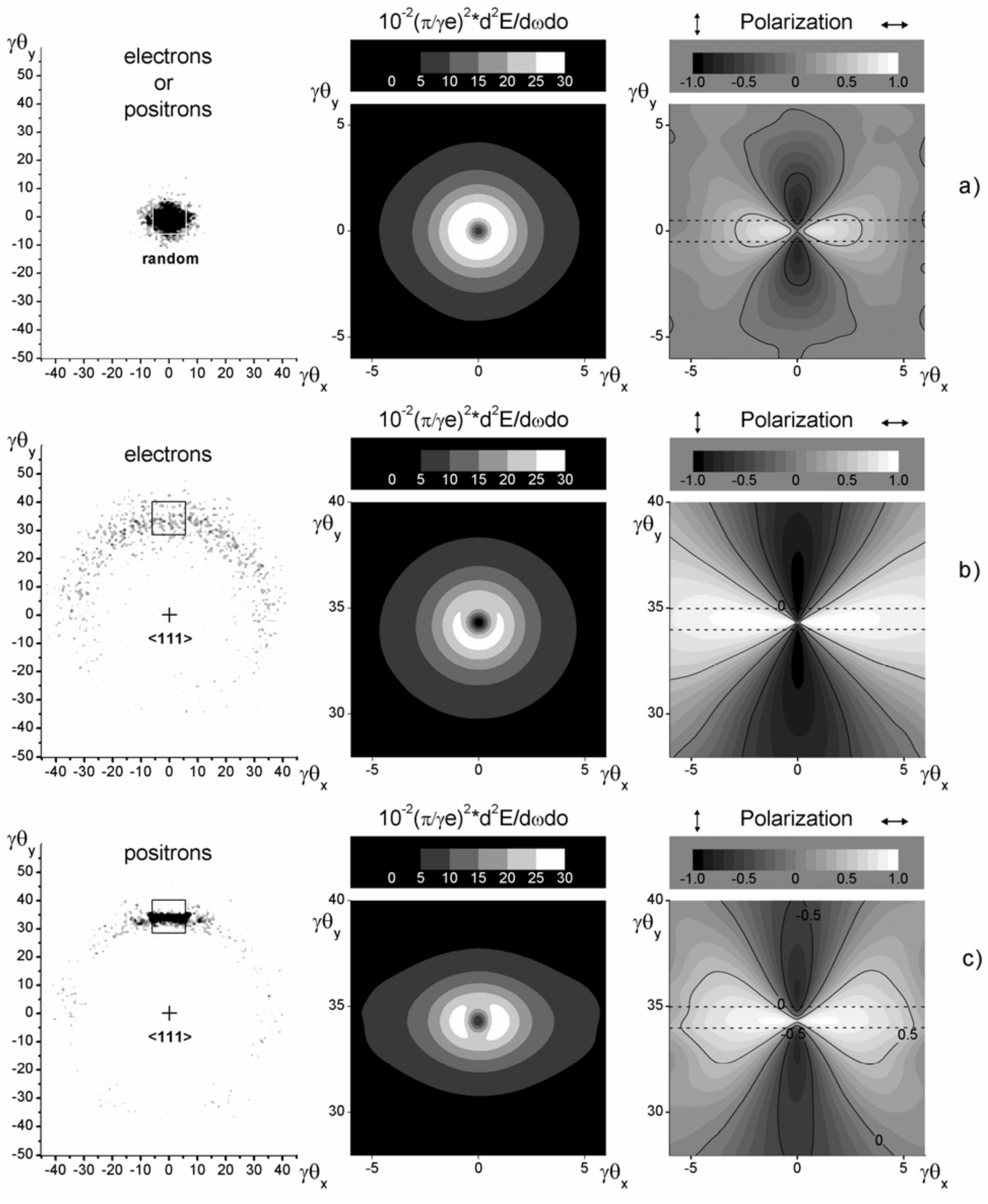

Fig.3. The angular distributions of scattered leptons (left) as a result of computer simulation of 200 GeV lepton beams propagation through a 20 μm tungsten crystal. Black crosses show the direction of the crystal axis <111>. The black square is the region that is represented in the middle part with corresponding zooming. The angular distributions of intensity (middle) and linear polarization degree (right) of radiation by scattered particles. Dashed lines shows the optimal position of the slit photon collimator with the angular width γ^{-1} to obtain the highest degree of linear polarization.

a) electrons or positrons scattered by crystal at random orientation (like in amorphous target);

b) electron beam is directed to the crystal at the angle $\psi = \psi_L$ to the axis <111>;

c) positron beam is directed to the crystal at the angle $\psi = \psi_L$ to the axis <111>

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ТОНКАЯ СТРУКТУРА УГЛОВЫХ РАСПРЕДЕЛЕНИЙ И ПОЛЯРИЗАЦИЯ ИЗЛУЧЕНИЯ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОНОВ И ПОЗИТРОНОВ В ТОНКОМ КРИСТАЛЛЕ

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Представлены результаты теоретического исследования угловых распределений и поляризации излучения релятивистских электронов и позитронов проходящих через тонкий кристалл под малым углом к оси кристалла. Показано, что нетривиальная тонкая структура угловых распределений излучаемых гамма-квантов обусловлена влиянием многократного рассеяния на излучение частиц высоких энергий в кристалле (аналогичный эффекту Ландау-Померанчука-Мигдала). Показано также, что используя щелевой коллиматор можно получить высокую степень линейной поляризации излучения, в то же время циркулярная поляризация излучения в тонком кристалле близка к нулю в любом направлении.

КЛЮЧЕВЫЕ СЛОВА: релятивистский электрон, релятивистский позитрон, гамма-излучение, недипольный режим, линейная поляризация, циркулярная поляризация, кристалл.