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SIMULATION OF THE GEV ELECTRONS COHERENT RADIATION IN ORIENTED CRYSTALS

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It is proposed a method of numerical simulation of the coherent radiation of electrons with energy ~GeV in oriented crystals. The method is based on semi-classical approximation of quantum electrodynamics and dipole approximation, and takes into account both coherent and incoherent effects at the particles scattering, effects of dechanneling and rechanneling at the particles passage through the crystal. The simulation results are in a good agreement with experimental data on radiation spectra of the electrons with energy ~GeV in silicon and diamond crystals for a wide range of the particles angles to the crystal axes.
KEY WORDS: simulation, oriented crystals, coherent radiation, semi classical model, dipole approximation, radiation spectra.

The motion of relativistic electrons in a crystal under small angles to one of the crystal axes has a rather complicated character that strongly influence on characteristics of their radiation (see [1,2]). Therefore, development of theoretical models and methods of the electron radiation calculation with taking into account their real dynamic in a crystal is of a large significance for correct description of the radiation phenomena. But accurate development of the problem is rather complicate and requires large computing resources for calculation. So, development of simple models allowing essentially accelerate the calculations with preservation adequate description of the real physical situation is of a big interest.

In present work one of the possible models is considered. It is based on the dipole approximation formulas of semi-classical theory of relativistic particles radiation in an external field [2,3]. The particles motion is considered on basis of the method developed earlier [4] for investigation of high energy particle passage through a bent crystal in which both coherent and incoherent effects of the particles scattering and effects of dechanneling and rechanneling have been taken into account. Results of the proposed model radiation spectra calculation are compared with experimental data [5] for electrons with energy ~ GeV.

SIMULATION OF RELATIVISTIC ELECTRON COHERENT RADIATION

When a high-energy electron moves in a crystal under small angle to one of the crystal axes the effective constant of the particle interaction with lattice atoms is large. Therefore in this case the motion and radiation of the electron can be considered within the framework of semi-classical approximation of quantum electrodynamics [2]. As is well known, in the semi-classical approximation the electron radiation cross section in an external inhomogeneous field can be represented through the particle classical trajectories in that field [2,6]. The trajectories are rather intricate and may correspond to various types of the particles motion relatively crystal atomic strings, both regular and random: axial and planar channeling, above barrier and chaotic motion.

For electrons with energy $\varepsilon \le 1$ GeV the condition of dipole radiation in a crystal is (ħ=c=1 system is used)

$$\gamma\vartheta_l << 1, \tag{1}$$

which is satisfied in the most cases of the electron interaction with crystals (especially for crystals of light elements). γ is the electron Lorentz-factor, ϑ_l is the electron scattering angle at the coherence length of the radiation process $l = 2\varepsilon\varepsilon'/m^2\omega$, $\varepsilon'=\varepsilon-\omega$, ε' and ω are the energies of the scattered electron and radiated photon, m is the electron mass. In the dipole approximation the semi-classical formula for radiation cross section is much simpler, that allows one to essentially simplify the simulation procedure of the particle radiation process. The spectral density of relativistic electron radiation intensity with taking into account the recoil effect in radiation is ([2,3])

$$\frac{dK}{d\omega} = \frac{e^2\omega}{4\pi}\int d^2\rho_0 \int_{\delta}^{\infty} \frac{dq}{q^2}\left[\frac{\varepsilon^2+\varepsilon'^2}{\varepsilon\varepsilon'} - 4\frac{\delta}{q}\left(1-\frac{\delta}{q}\right)\right] \left|\mathbf{W}(q)\right|^2, \tag{2}$$

$\delta=l^{-1}$, $\mathbf{W}(q)$ is the Fourier-component of acceleration in orthogonal direction to the velocity of the incident electron **v**,

$$\mathbf{W}(q) = \int_{-\infty}^{\infty} dt \dot{\mathbf{v}}_{\perp}(t, \boldsymbol{\rho}_0) e^{iqt}, \tag{3}$$

$\boldsymbol{\rho}_0$ is the transversal coordinate (impact parameter) at which the electron enters the crystal. Thus, the intensity of the electron radiation (2) is determined by the particle trajectory in a crystal in the plane orthogonal to **v**.

At small angles ψ between the electron momentum and one of the crystal axes (*z* axis) the particle motion in the crystal is mainly determined by the lattice potential averaged over *z* coordinate [2,7,8]

$$U_c(\boldsymbol{\rho}) = \frac{1}{L_z} \sum_n \int_{-\infty}^{+\infty} dz U(\boldsymbol{\rho} - \boldsymbol{\rho}_n, z - z_n), \tag{4}$$

where L_z is the crystal thickness along *z* axis, $U(\boldsymbol{\rho} - \boldsymbol{\rho}_n, z - z_n)$ is the potential of lattice atom placed in the point with coordinates $\mathbf{r}_n = (\boldsymbol{\rho}_n^0 + \mathbf{u}_{\perp,n}, z_n^0 + u_{z,n})$, $(\boldsymbol{\rho}_n^0, z_n^0)$ are the equilibrium positions of the atoms in the lattice and $\mathbf{u}_n$ are their thermal displacements. The summation in (4) runs over all crystal atoms. The trajectory in the orthogonal to *z* axis plane (*x*, *y*) of the crystal is determined by the equation

$$\ddot{\boldsymbol{\rho}} = -\frac{e}{\varepsilon} \frac{\partial}{\partial \boldsymbol{\rho}} U_c(\boldsymbol{\rho}). \tag{5}$$

The thermal displacements of the atomic positions in a lattice are random quantities; therefore the particle trajectory in the crystal is also random one. Thus the radiation intensity (2) should be averaged over all the particle trajectories in the crystal. In the simplest case, when the electron trajectory in the (*x*,*y*) plane is close to rectilinear such averaging can be carried out in general case. Indeed, the close to rectilinear movement takes place if the particles enter the crystal under angles ψ (to the crystal string) and θ (to the crystal planes) considerably exceeding the critical angles of the axial ψ_c and planar θ_c channeling, respectively. In this case one may find the solution of the equation (5) as a series expansion of the potential. In the first non-vanishing approximation after averaging over thermal oscillations of lattice atoms the formula (2) gives the result that coincides with the corresponding result of the theory of relativistic electrons coherent radiation in a crystal [2,9]. At $\psi \sim \psi_c$ the electron trajectories in crystal become very intricate, their motion can obtain the regular or chaotic character [1,10] and the computation procedure becomes more difficult. Thermal oscillations of lattice atoms also strongly influence on the electron motion.

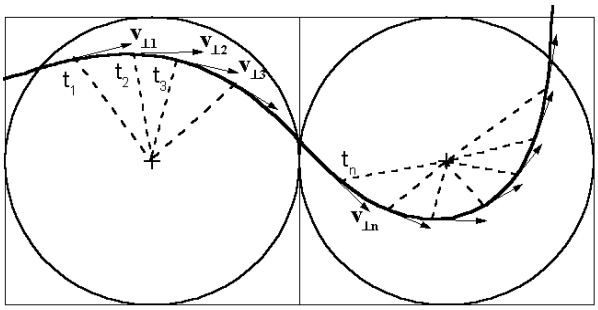

Fig. 1. The partition of a particle trajectory in the (x,y) plane

For calculation of the Fourier component $\mathbf{W}(q)$ of the electron acceleration total time of it interaction with the crystal was divided into small intervals Δt, Fig. 1, at which the integrand in (3) was weakly changed. Then the quantity $\mathbf{W}(q)$ can be represented as a sum

$$\mathbf{W}(q) = \sum_n e^{iqt_n} \Delta \boldsymbol{\theta}_n, \tag{6}$$

where $\Delta \boldsymbol{\theta}_n = \Delta t \dot{\mathbf{v}}_{\perp n} = \mathbf{v}_{\perp n} - \mathbf{v}_{\perp n-1}$ is the electron velocity change at the *n*-th part of the trajectory and t_n is the time, when the n-th part of the trajectory begins. Besides, the time intervals Δt were chosen in such a way, that the longitudinal parts of the trajectories passed by the particle were small in comparison with the radiation coherence length l_c. These requirements can always be satisfied if the number of the trajectory partitions in the transversal (x,y) plane is large enough. At present work the electron trajectory was divided into 20 parts within one elementary crystal cell.

For calculation of the $\Delta \boldsymbol{\theta}_n$ and t_n values numerical simulation program was used [4] developed earlier for the fast charged particle passage description through a bent crystal near one of the crystal axes. The particle trajectory calculation is carried out in this program by step-by-step solution of the particle motion equation in the field of continuous potential of the crystal strings averaged over the lattice atoms thermal oscillations. The incoherent effects in scattering are taken into account through fluctuations of the particle velocity, which are included at every part of the trajectory through the lattice potential fluctuations, caused by the thermal spread of the crystal atoms. Continuous potential of a crystal (4) is taken in the form

$$U_c(\boldsymbol{\rho}) = \bar{U}_c(\boldsymbol{\rho}) + \delta U_c, \tag{7}$$

where $\bar{U}_c(\boldsymbol{\rho})$ is the potential (4) averaged over thermal oscillations of the lattice atoms, δU_c is the fluctuation of the

potential due to the specific choice of thermal atom displacements $\mathbf{u}_n$. At first equation (5) is solved for each part of the trajectory under assumption $\delta U_c = 0$. Then at the end of the each trajectory part the value of the electron velocity, acquired due to scattering at the potential δU_c fluctuations, is picked out at random. On the basis of the sets of $\Delta\boldsymbol{\theta}_n$ and t_n values calculated for each trajectory the quantity (6) was determined and then the formula (2) was averaged over trajectories. The simulation of the coherent radiation of relativistic electrons in a crystal was also done in [11,12], however, the influence of incoherent effects in scattering on radiation was not taken into account in these papers.

COMPARISON WITH EXPERIMENT

Approbation of the proposed model was carried out by comparing the calculation results with experimental data on radiation spectra of 1.2 GeV electrons in silicon and diamond crystals obtained earlier at Kharkov Institute of Physics and Technology and presented in [5,13,14], where details of the experimental equipment and procedure were described. The electron beam divergence did not exceed 10^{-4} rad in these experiments.

The presented below radiation spectra for silicon crystals were measured both for electron angle of incident on crystal axis ψ=0 (axial orientation, the axial channeling may be realized) and for the angles ψ≥ψ$_c$. In the last case the above barrier particles motion dominates and the channeling may be realized only due to particle rechanneling. In Fig. 2 the radiation spectra of electrons in silicon crystal 0.07 mm thickness are presented, when they fly into the crystal along <111> axis (ψ=0) and under the angle ψ=2.5ψ$_c$ to this one. The data obtained for more thick silicon crystal (0.24 mm) for angles of incident ψ=0 and ψ=1.2ψ$_c$ on the crystal axis<110> are plotted in Fig. 3. The critical angle of the axial channeling is ψ$_c$~0.4 mrad for these experimental conditions.

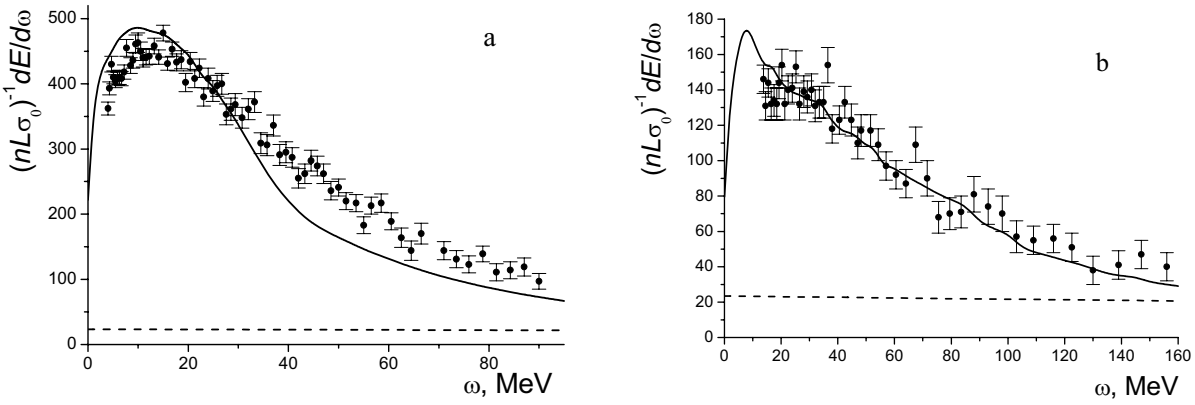

Fig. 2. Radiation spectra of the electrons with energy ε=1.2 GeV moving in silicon crystal 0.07 mm under angle ψ to <111> axis. a - ψ= 0; b - ψ≈ 2.5ψ$_c$. Solid lines are results of the simulation, dashed lines are the Bethe-Heitler spectrum; L is the target thickness, n is the atomic density, $\sigma_0 = Z^2 e^6 m^{-2}$.

One can see that in the case of axial orientation the radiation spectra contain a broad maximum in the photon energy region 15…20 MeV. The radiation intensity in this region is ~ 20-25 times larger than the radiation intensity from disoriented crystal for 0.07 mm, Fig.2a, and somewhat less for more thick crystal, Fig. 3a. When the crystal 0.24 mm is turned such a way that electrons fly into crystal under angle ψ=1.2ψ$_c$ to crystal axis the photon spectra does not practically change, Fig 3b. This indicates that both for ψ=0 and ψ=1.2ψ$_c$ the main mechanism of particle radiation in the crystal in this energy range is mainly the coherent radiation of above-barrier particles, i.e. particles performed infinite motion relatively crystal atomic strings. Notice, that for disorientation of the silicon crystal 0.07 mm thick at the angle ψ~2ψ$_c$ the radiation intensity decreases in 2…3 times, Fig. 2b. It means that for thin crystal at axial orientation more significant contribution to radiation comes from the channeling electrons, whereas for more thick crystals the contribution of such particles into radiation is not practically important even for ψ=0. Such modification of the radiation mechanism with increasing of the crystal thickness can be attributed to the particle dechanneling. As it has been shown from analysis of nuclear reactions yield depending on the depth of ~1 GeV electron penetration into the crystal [15] the crystal dechanneling length is of some tens of micrometers. The simulation results as a whole is in a rather good agreement with the experimental data with the exception of low energy region of the spectra for crystal 0.24 mm thick, Fig. 3. The some disagreements between theory and experiment at axial orientation in the energy range 40…70 MeV (Fig. 2a, 3a) might be produced by superposition of the detectors signal due to multiple low energy photon production.

The above results are concerned to the case, when the crystal was oriented in such a way that electron moved under angle $\psi \sim \psi_c$ to crystal axis but far from densely packed crystal planes. Results for other type of crystal orientation, when the electrons move under angle $\psi >> \psi_c$ to the crystal axis but at the same time along the densely packed crystal plane are shown in Fig. 4. In this case two maxima in radiation spectra are appeared: one is at energy $\omega \sim 40$ MeV, the second one in the region of several hundreds MeV. Position of the latter peak changes with changing the ψ, whereas position of low energy maximum and its magnitude do not depend on the ψ variation. It means that the peak at low energy does not depend on the plane structure. On the contrary the high energy peak depends on the plane structure and moreover it is resulted due to interference of the radiation from the electron interaction with atomic strings periodically spaced out in the plane. So, these peaks are the channeling radiation peak and coherent bremsstrahlung one and they correspond to two periods of particle motion: a large period related to periodical motion in the planar potential and a small period related to electron collision with the atomic strings.

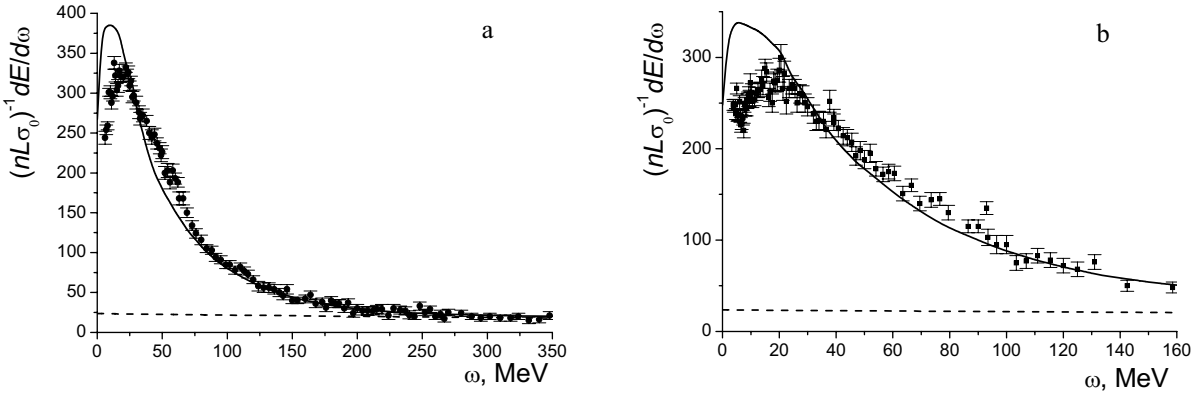

Fig. 3. Radiation spectra of the electrons with energy ε=1.2 GeV moving in silicon crystal 0.24 mm under angle ψ to <110> axis. a - ψ= 0; b - ψ≈ $1.2\psi_c$. Solid lines are results of the simulation, dashed lines are the Bethe-Heitler spectrum; L is the target thickness, n is the atomic density, $\sigma_0 = Z^2 e^6 m^{-2}$.

Solid curves in Fig. 4 are results of the calculations. On the whole there is also a rather good agreement between calculations and experiment with the exception of the low energy range of the spectra, as in Fig. 3. These discrepancies may be due to more complicate character of the motion of the particles radiate in this spectral range that requires the more accurate simulation procedure. Dash-dotted curves in Fig. 4 are the calculations in the Born theory of the coherent radiation of relativistic electron in a crystal [2,9]. Significant disagreement of the Born theory calculations with experimental data and the calculations in the proposed model are resulted from the fact that Born theory does not take into account real particle dynamics in a crystal, whereas as one can see the radiation spectra essentially depend on the character of the particle motion in a crystal.

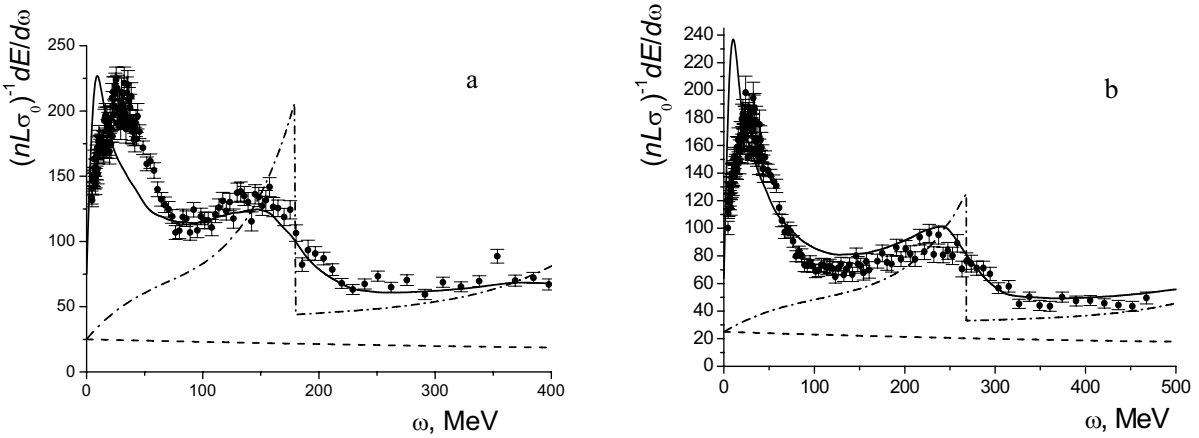

Fig. 4. Radiation spectra of the electrons with energy ε=1.2 GeV moving in diamond crystal 0.3 mm along the plane (110) under angle ψ to <110> axis. a - ψ= 5.5 mrad; b- ψ≈ 9 mrad.
Solid lines are results of the simulation, dashed lines are the Bethe-Heitler spectrum, dash-dotted lines are the calculations in the Born theory of the coherent radiation of relativistic electron in a crystal [2,9]. L is the target thickness, n is the atomic density, $\sigma_0 = Z^2 e^6 m^{-2}$.

CONCLUSIONS

It is described the method of numerical simulation of the coherent radiation of electrons with energy ~GeV in oriented crystals which is based on semi-classical approximation of quantum electrodynamics and dipole approximation, and takes into account both coherent and incoherent effects at the particles scattering, effects of dechanneling and rechanneling at the particles passage through the crystal. Results of the simulation are in a reasonable agreement with experimental data on radiation spectra of the electrons with energy ~GeV in silicon and diamond crystals for a wide range of the particles angles to the crystal axes, so the proposed model reflects the real physical picture of interaction of GeV electron with crystals.

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МОДЕЛИРОВАНИЕ КОГЕРЕНТНОГО ИЗЛУЧЕНИЯ ЭЛЕКТРОНОВ С ЭНЕРГИЕЙ ПОРЯДКА НЕСКОЛЬКИХ ГИГАЭЛЕКТРОН-ВОЛЬТ В ОРИЕНТИРОВАННЫХ КРИСТАЛЛАХ

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Предложен метод численного моделирования когерентного излучения электронов с энергией ~ГэВ в ориентированных кристаллах. Он основан на квазиклассической модели и дипольном приближении и учитывает эффекты когерентного и некогерентного рассеяния, эффекты деканалировния и реканалирования при движении частиц в кристалле. Результаты моделирования согласуются с экспериментальными спектрами излучения электронов в кристаллах кремния и алмаза.

КЛЮЧЕВЫЕ СЛОВА: моделирование, ориентированные кристаллы, когерентное излучение, квазиклассическая модель, дипольное приближение, спектры излучения.