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STUDY OF THE SYNCHROTRON DYNAMICS IN COMPTON X-RAY RING

E.V. Bulyak, P.I. Gladkikh, V.V. Skomorokhov
National Science Center "Kharkov Institute of Physics and Technology", Ukraine, Kharkov, 61108, Akademicheskaya 1
E-mail: scomvv@kipt.kharkov.ua
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Within the bounds of a model of the electron storage ring that consist only of a drift and a radio-frequency cavity, the longitudinal dynamics of electron bunches with large energy spread is considered. Under consideration, the model of the storage ring with small momentum compaction factor is sufficiently nonlinear. The finite-difference equations and differential equations of motion in the canonical form are derived for this model. Based on the differential model, topology of the longitudinal phase space and its modifications due to changes in the ring parameters is considered. The analytical expression for the factor, which determined the topology of the longitudinal phase space, was derived. The response of equilibrium area upon changes of the nonlinear momentum compaction factor is presented. It is show that enlargement of the energy acceptance of the ring by decreasing of the momentum compaction factor is limited with the nonlinearity in the compaction factor. Decreasing of the linear compaction factor below the certain limit causes the reversed effect – decreasing of the acceptance. Comparative analysis of the finite-difference and differential models is made by means of computational simulations. The results of simulation manifest a good agreement with the theoretical predictions on the sizes and the positions of equilibrium areas.
KEY WORDS: electron storage rings, synchrotron nonlinear dynamics, Compton X-ray sources

Engagement of electron storage rings for production of x-rays through Compton scattering of laser photons against ultrarelativistic electrons was proposed in 1998 [1]. Two basic schemes exist so far. One of them supposes use of electron beams with unsteady-state parameters [2] and applies the continual injection (and ejection of circulating bunches by the next injecting pulse) of dense intensive bunches. The second scheme is based on the continuous circulation of bunches. For keeping of the bunches with a sufficiently large energy spread (see [3]), a lattice with small controllable momentum compaction factor is proposed to employ [4]. Longitudinal dynamics in the small momentum compaction factor lattice is governed not only by the linear effects of the momentum deviation but by the nonlinear ones as well.

In Compton sources storing the bunches with the large energy spread which can be as high as a few percents, ring's energy acceptance becomes comparable to the energy spread. To achieve proper lifetime of the circulating electrons, the energy acceptance $\sigma \equiv \max (E-E_s)/E_s$ (E_s is the energy of a synchronous particle) should be high enough.

Within linear approximation according to the energy deviation, the acceptance can be increased either by enhancement of the radio frequency (RF) voltage, V_{RF}, or by decreasing of the linear momentum compaction factor α_0 since $\sigma \propto (V_{RF}/\alpha_0)^{1/2}$.

The paper presents result of study on the longitudinal dynamics of electron bunches circulating in storage rings with small linear momentum compaction factor α_0. Structure of the phase space and its modification with changes in the ring lattice parameters is considered. In particular, the size of stable area as a function of the RF voltage and momentum compaction is evaluated.

FINITE-DIFFERENCE MODEL

Let us consider a model of the ring comprised only two components: a drift and an RF cavity. For the sake of simplicity, we will suggest the cavity infinitely short, in which the particle momentum (energy) suffers an abrupt change while the phase of a particle remains unchanged. On the contrary, the phase of a particle traveling along the drift is changed while the energy remains invariable. The longitudinal motion in such idealized ring will be described in the canonically conjugated variables ϕ (the phase about zero voltage in the cavity) and the momentum $p \equiv (\gamma-\gamma_s)/\gamma_s$ equal to the relative deviation of the particle energy from the synchronous one (γ_s is the Lorentz factor of synchronous particle).

To study a system able to keep the beams with large energy spread, one need to take into account not only the linear part of the orbit deviation from the synchronous one, but nonlinear terms as well:

$$\Delta x \approx D_1 p + D_2 p^2 + \ldots,$$

where D_1 and D_2 are the dispersion functions of the first and the second orders, respectively.

Accordingly, relative lengthening of a flat orbit is

$$\frac{\Delta L}{L_0} = \oint \sqrt{\left(1+\frac{\Delta x}{\rho}\right)^2 + \left(\frac{d\Delta x}{ds}\right)^2} ds \approx \alpha_0 p + \alpha_1 p^2 + \ldots,$$

where L_0 is the length of synchronous orbit, $\rho(s)$ is the local radius of curvature, s is the longitudinal coordinate. The coefficients α_0 and α_1 are determined as

$$\alpha_0 = \frac{1}{L_0}\oint \frac{D_1}{\rho} ds\,; \tag{1}$$

$$\alpha_1 = \frac{1}{L_0}\oint \left(\frac{D_1'^2}{2} + \frac{D_2}{\rho} \right) ds\,. \tag{2}$$

In accordance with the definitions of α_0 and α_1, the momentum compaction factor α_c can be written as

$$\alpha_c = \frac{1}{L_0}\frac{dL}{dp} \approx \alpha_0 + 2\alpha_1 p + \dots . \tag{3}$$

To study the phase dynamics in a storage ring with the small momentum compaction factor α_0, the next terms of expansion of the compaction over the energy deviation should be accounted for, hence — higher terms in the sliding factor η [5-7]. The sliding factor η is defined according to relation

$$\frac{\Delta\phi}{\phi} = \eta(p)p \approx (\eta_0 + \eta_1 p + \ldots)p\,, \tag{4}$$

with η_0 and η_1 determined by

$$\eta_0 = \alpha_0 - \frac{1}{\gamma_s^2}\,; \tag{5}$$

$$\eta_1 = \alpha_1 - \frac{\alpha_0}{\gamma_s^2} + \frac{3}{2\gamma_s^2}\,. \tag{6}$$

Consequently, the factor η characterizes a relative variation of the phase due to changes of the particle velocity and orbit length.

The finite-difference equations for the phase ϕ and the variation of relative energy p in the model under consideration read

$$\phi_f = \phi_i + \left(\kappa_0 p_i + \kappa_1 p_i^2\right)\Delta\tau\,; \tag{7}$$

$$p_f = p_i - U_{rf}\sin\phi_f \Delta\tau\,, \tag{8}$$

$$U_{RF} = \frac{eV_{RF}}{\gamma_s E_0}\,,$$

where V_{RF} is the RF-voltage, $E_0=mc^2$ is the rest energy; and

$$\Delta\tau = \tau_f - \tau_i = \frac{\beta c}{L}\left(t_f - t_i\right),$$

the subscripts i and f correspond to the initial and final values, respectively. The dimensionless variable $\tau = t\beta c/L$ represents time expressed in number of rotations (t is time, βc the velocity of a particle). The factors κ_0 and κ_1 at large γ_s are determined by the expressions $\kappa_0 = 2\pi h\eta_0 \approx 2\pi h\alpha_0$, $\kappa_1 = 2\pi h\eta_1 \approx 2\pi h\alpha_1$ (h is the harmonic number).

From Eqs. (7), (8) differential (smoothed) equations can be deduced. As it is seen, the equation (8) contains the final value of the phase ϕ_f expressed via the initial value ϕ_i and momentum p_i by the equation (7).

Let us expand $\sin\phi_f$ into series of powers $\Delta\tau$:

$$\sin\phi_f = \sin\left(\phi_i + \left(\kappa_0 p_i + \kappa_1 p_i^2\right)\Delta\tau\right) \approx \sin\phi_i + \cos\phi_i(\kappa_0 p_i + \kappa_1 p_i^2)\Delta\tau\,.$$

Since $\Delta\tau$ cannot be regarded as infinitesimal (formally Eqs. (7),(8) present a full turn, $\Delta\tau = 1$), then the linear term can be neglected if $\kappa_0 p_i + \kappa_1 p_i^2 \ll 1$. In the considered case, it can be done since maximum of the energy spread does not exceed a few percents, and the momentum compaction factor α_0 supposed small. Under these assumptions, the finite difference equations can be reduced to

$$\frac{\Delta\phi}{\Delta\tau} = \kappa_0 p_i + \kappa_1 p_i^2\,; \tag{9}$$

$$\frac{\Delta p}{\Delta \tau} = -U_{RF} \sin \phi_i \,. \tag{10}$$

DIFFERENTIAL MODEL

Noting of formal similarity of Eq. (8) to the canonical Hamilton equations describing the mathematical pendulum, we can use a smoothed analog to these equations (a differential substitute for a finite-difference equation, $\Delta\tau \to 0$) to facilitate analysis of the motion

$$\frac{d\phi}{d\tau} = \kappa_0 p + \kappa_1 p^2 \,; \tag{11}$$

$$\frac{dp}{d\tau} = -U_{RF} \sin \phi \,. \tag{12}$$

A Hamilton function for (11) and (12) possesses a specific form with the cubic canonical momentum term

$$H = \frac{\kappa_1}{3} p^3 + \frac{\kappa_0}{2} p^2 - U_{RF} \cos \phi \,. \tag{13}$$

Phase portraits of motion with the Hamiltonian (13) are represented in Fig. 1. The magnitude and sign of the parameter

$$\mu = 2\sqrt{3\frac{\kappa_1^2}{\kappa_0^3} U_{RF}} = \sqrt{\frac{6\alpha_1^2 e V_{RF}}{\pi h \alpha_0^3 \gamma_s E_0}} \tag{14}$$

govern topology of the phase plane.

At zero value (μ=0 that is κ_1=0), the Hamiltonian (13) has a form of the mathematical pendulum; its phase plane is presented in Fig.1(a). Within the interval $0 \le \mu^2 < 1$, there an additional area of finite motion appears; this area is separated from the main area with the band of infinite motion as depicted in Fig. 1(b). When the parameter μ exceeds the critical value $\mu_c^2 = 1$ [see Fig. 1(c)], e.g. $1 \le \mu < \infty$, the structure of the phase plane will have changed as is represented in Fig. 1(d).

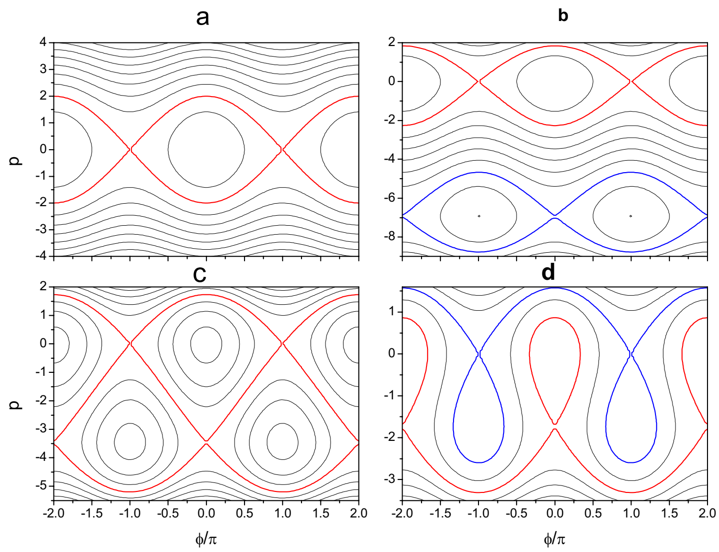


Fig.1. Phase portrait of longitudinal motion with account for the cubic nonlinearity at different values of the parameter μ: μ=0 (a), $\mu<\mu_c$ (b), $\mu=\mu_c$ (c), $\mu>\mu_c$ (d).

The dimension of a stable (finite) longitudinal motion, i.e., the area comprised by a separatrix, is in direct proportion with ratio of the ring parameters. For the considered case of the nonlinear Hamiltonian (13), the separatrix height (size along the *p* axis) is determined by

$$\Delta p = \frac{\alpha_0}{\alpha_1}\left(\cos\frac{\xi}{3} + \cos\left(\frac{\xi}{3} + \frac{\pi}{3}\right)\right); \tag{15}$$

$$\cos \xi = 12 U_{RF} \frac{\alpha_1^2}{\pi h \alpha_0^3} - 1 \,;$$

$$\Delta p = \frac{3}{2}\frac{\alpha_0}{\alpha_1} \,, \tag{16}$$

for $\mu \le \mu_c$ (15) and $\mu \ge \mu_c$ (16), respectively.

The phase width of the separatrix (dimension along the ϕ axis) is determined by expressions

$$\Delta\phi = 2\pi \; ; \tag{17}$$

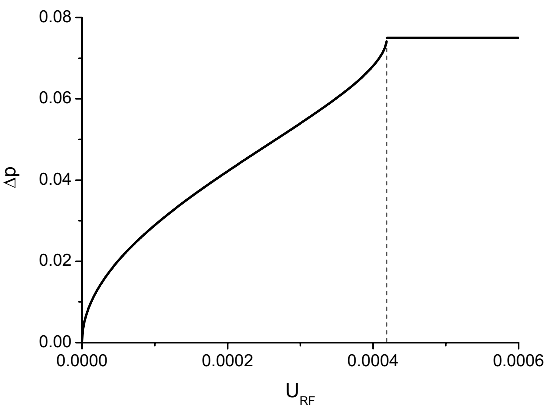
$$\Delta\phi = 2\arccos\left(1 - \frac{\pi h}{3U_{RF}}\frac{\alpha_0^3}{\alpha_1^2}\right), \tag{18}$$

for the subcritical (17) and the overcritical (18) values of the parameter μ, respectively.

Dependence of the phase and momentum separatrix dimensions on RF amplitude at the fixed other parameters, which values are listed in Table, is presented in Fig. 2.

Table. Ring parameters

Parameter	Designation	Value
Acceleration Voltage	V_{RF}	$4\Box 10^5$ V
Lorentz factor	γ_s	84
Harmonic number	h	32
Linear compaction factor	α_0	0.01
Quadratic compaction factor	α_1	0.2



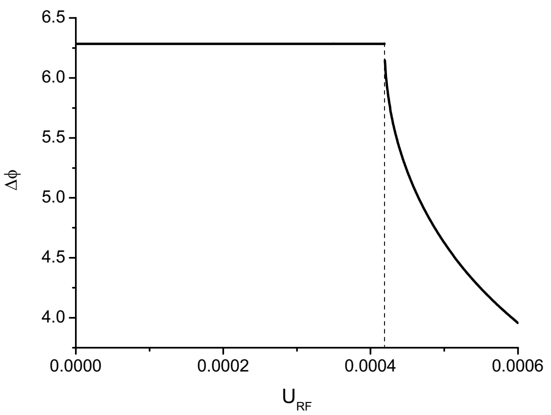


Fig. 2. Separatrix height (above) and width (below) as functions of the parameter U_{RF}.

As it can be seen from the plot in Fig. 2, while increasing the parameter U_{RF}, the separatrix height grows reaching its maximum, $\Delta p \approx 7.5\times10^{-2}$, at $U_{RF(c)} \approx 4.1\times10^{-4}$ (which is equal to the RF voltage of $V_{RF(c)} \approx 17.560$ kV at γ_s=84).

With further increase in the RF voltage, the separatrix height remains constant. The separatrix width remains constant with increase of the RF voltage up to the critical value U_{RF}, and then it is diminishing.

In Fig. 3, dependence of the separatrix dimensions on the linear momentum compaction factor under other system parameters fixed is presented.

Quite the reverse to the dependence $\Delta\phi = \Delta\phi(U_{RF})$, a dependence of the separatrix width upon the linear compaction factor, $\Delta\phi = \Delta\phi(\alpha_0)$, is increasing while α_0 grows. At a certain critical value of the linear momentum compaction factor $\alpha_{0(c)}$ (in the considered case $\alpha_{0(c)} \approx 2.8\times10^{-2}$), the width of equilibrium area has reached its maximum and remains constant with further increase in α_0. A dependence of the separatrix height on α_0 is of increasing within interval $0 \le \alpha_0 \le \alpha_{0(c)}$. Then, after the maximum at $\alpha_0 = \alpha_{0(c)}$ this dependence becomes declining, coming to zero at the large α_0.

Since the phase volume enclosed within the separatrix (and, therefore, the storage ring acceptance) is proportional to product of the transverse dimensions of the separatrix, $\sigma \sim \Delta p\Delta\phi$, then from comparison of the plots in Fig. 2 and Fig. 3 it follows that optimal working point is about the critical parameters.

In addition, it can be seen that, unlike a linear lattice, the nonlinear terms in the momentum compaction factor restrict the infinite increase of the energy acceptance with decreasing of the linear momentum compaction factor. The acceptance increase takes place while the linear compaction is above certain critical value $\alpha_{0(c)}$, which is determined by the ring lattice parameters according to equation

$$\frac{6\alpha_1^2 eV_{rf}}{\pi h\alpha_0^3\gamma_s E_0} = 1 \,. \tag{19}$$

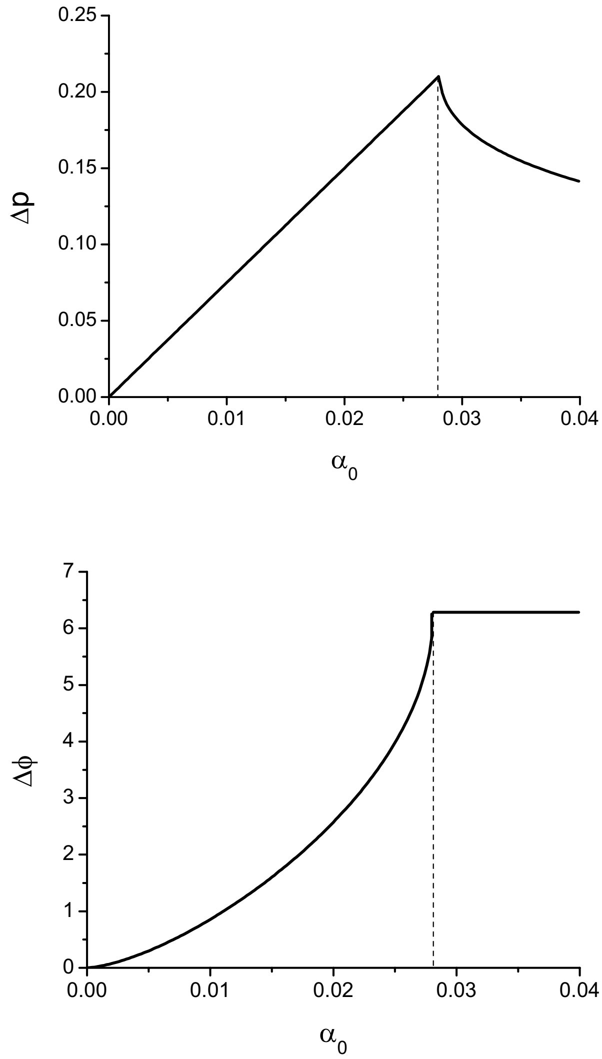


Fig. 3. Separatrix height (top) and width (bottom) as a function of α_0.

With further decrease of α_0 the acceptance also decreases.

To validate usage of the differential (smoothed) equations of motion (11) for analysis of Compton storage ring, a code has been developed based on the finite difference equations (7). A simulated phase-plane portrait for the ring parameters listed in Tab.1 for $\mu \geq \mu_c$ is presented in Fig.4.

From the figure it follows that the electrons can be confined within not only the "linear" area (minimum of Hamilton function (13)), but also the "nonlinear" as well. (The nonlinear stable region disappears in the linear lattice.) RMS sizes and the center of weight positions perfectly correspond to the analytical estimations presented above.

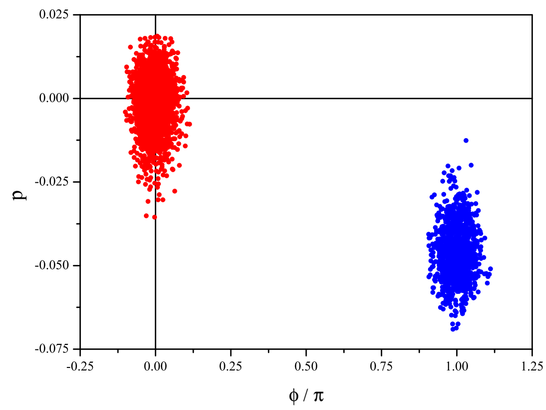


Fig. 4. Distribution of kept electrons over the longitudinal phase plane in a system with cubic nonlinearity at $\mu \geq \mu_c$;
left bunch corresponds to "linear" case, right - to "nonlinear" (additional).

SUMMARY

Results of the study on dynamics of synchrotron motion of particles in the storage rings with the nonlinear momentum compaction factor presented in the paper can be digested as follows:

- Grounded on a simplified model of the storage ring, the finite-difference equations were derived. Hamiltonian treatment of the phase space structure was performed. As was shown, the structure of the phase space is governed by ratios of the ring parameters. The analytical expression for the factor μ, which determined the topology of the longitudinal phase space, was derived.
- Dependencies of the sizes of the equilibrium areas of the synchrotron motion in a nonlinear lattice were derived. Analysis of dependence of the longitudinal acceptance upon the amplitude of RF voltage and the linear compaction factor at the fixed quadratic nonlinear term was presented. As was shown, the acceptance is growing up only to a definite magnitude determined by the critical value of parameter $\mu = \mu_c$. It was emphasized that in order to maximize the acceptance of the lattice with the small linear momentum compaction factor and the wide energy spread of electrons in the bunches, the system parameters should be chosen close to the critical value of μ.
- To validate usage of the smoothed equations of motion, the simulating code was developed. The code is based on

the finite-difference equations. The results of simulation manifest a good agreement with the theoretical predictions on the sizes and the positions of equilibrium areas.

The results obtained allow one to make the following conclusion: Enlargement of the energy acceptance of the ring by decreasing of the momentum compaction factor is limited with the nonlinearity in the compaction factor. Decreasing of the linear compaction factor below the certain limit causes the reversed effect – decreasing of the acceptance.

Similar consequence corresponds to the build-up of the RF voltage: Increase of the voltage above a certain limit causes narrowing of possible bunch length while the energy acceptance remains constant. This effect can lead to decrease in the injection efficiency for high RF voltages.

REFERENCES

1. Z. Huang, R.D. Ruth Laser-electron storage ring // Phys. Rev. Lett.-1998.-V. 80.-P. 976-979.
2. R.J. Loewen A compact light source: design and technical feasibility study. Ph.D. thesis, Stanford 2003.
3. P.I. Gladkikh Lattice and beam parameters of compact intense X-ray source based on Compton scattering // Phys. Rev. ST Accel Beams.-2005.-V. 8. 050702.
4. E.V. Bulyak Laser cooling of electron bunches in Compton storage rings. Proc. EPAC-2004 http://accelconf.web.CERN.ch/accelconf/e04/papers/.
5. C. Pellegrini, D. Robin Quasiisochronous storage ring // NIM.-1991.-V A301.-P. 27-36.
6. Liu Lin, E.T. Gonsalves da Silva // Second order single particle dynamics in quasiisochronous storage rings and its applications to the LNLS-UVX ring // NIM.-1993.-V A329.-P. 9-15.
7. J. Feikes et al. The BESSY low alpha optics and the generation of coherent synchrotron radiation // ICFA Beam dynamics newsletter.-2004.-V 35.-P. 82-95.

ИССЛЕДОВАНИЕ СИНХРОТРОННОЙ ДИНАМИКИ В КОМПТОНОВСКОМ ИСТОЧНИКЕ

Е.В. Буляк, П.И. Гладких, В.В. Скоморохов

Национальный Научный Центр "Харьковский Физико-технический Институт",

Украина, Харьков, 61108, Академическая 1

E-mail: scomvv@kipt.kharkov.ua

В рамках модели электронного накопительного кольца, состоящего только из ВЧ-резонатора и дрейфового промежутка, исследуется продольная динамика электронных сгустков с большим энергетическим разбросом. При этом рассмотренная модель накопителя существенно не линейна и имеет малый линейный коэффициент уплотнения орбит. Для данной модели кольца построены конечно-разностные уравнения и дифференциальные уравнения движения в канонической форме. На основе дифференциальной модели исследована топология продольного фазового пространства и ее изменение в зависимости от соотношения параметров накопительного кольца. Также получено выражение для фактора, который определяет структуру продольного фазового пространства. Исследована зависимость величины области устойчивости пучка от величины и соотношения параметров системы. Показано, что увеличение энергетического акцептанса накопителя при уменьшении линейного коэффициента упаковки орбит ограничено наличием нелинейности. Уменьшая линейный коэффициент упаковки орбит ниже некоторого предельного значения, получаем обратный эффект – уменьшение акцептанса. На основе численного моделирования проведен сравнительный анализ конечно-разностной и дифференциальной моделей рассматриваемого накопителя. Результаты моделирования демонстрируют хорошее совпадение с теоретическими расчетами размеров и положения областей устойчивости.

КЛЮЧЕВЫЕ СЛОВА: накопительные электронные кольца, нелинейная синхротронная динамика, комптоновские рентгеновские источники