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IMPURITY ION TRANSPORT UNDER THE EFFECT OF DRIFT-WAVE-LIKE ELECTROSTATIC FIELD

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Heavy impurity ion transport is considered in the vicinity of adjacent rational magnetic surfaces under the drift-wave-like potential. Analytical study is done for the simplified magnetic field in the toroidal magnetic trap with the rotational transform of the magnetic field lines. The numerical study is carried out for the helical plasma. The test particle orbits are followed by the numerical integration of Newton-Lorentz equation system for the charged particle in the magnetic field and quasi-static electric field. As the test particle the tungsten ion is taken.
KEY WORDS: drift wave electric field, electrostatic islands, particle trapping process, impurity ion transport, helical magnetic field

We would like to emphasize that the drift-wave-like electrostatic field can be helpful to remove the impurity ions from the periphery of the magnetic confinement volume to outside. The physics mechanism of such transport of heavy ions is rather similar to that one found in [1] for the high energy ions, so called, cold α -particles and named as the estafette of drift resonances. Let us remind the essence of this mechanism. The trajectories of the passing particles can form the rational drift surfaces. If there are some adjacent rational drift surfaces with the drift rotational transform $\iota^* = n/m$, $\iota^* = n'/m'$, $\iota^* = n''/m''$, then the magnetic perturbations with the wave numbers (m,n), (m',n'), (m'',n'') can lead to some families of drift islands. Overlapping of the adjacent resonance structure is the reason for the stochasticity of the particle trajectories. If a particle trajectory passes through the set of perturbations this test particle can escape from the center of the confinement volume to the periphery. The helically trapped particles with the orbits of the helical banana-like can be transferred into the "toroidally trapped" particles under the effect of this electromagnetic field.

Very important fact found here is the following: the similar effect can take place under the electrostatic field effect if the electrostatic field potential has the form $\tilde{\Phi} = \sum_m \tilde{\Phi}_{m,n,0}(k_m r)\cos[m\vartheta - n\varphi - \eta(t)]$ or some different form but rather close to this one. Here r,ϑ,φ are the coordinates connected with the circular axis of torus; m and n are the poloidal and toroidal mode numbers; $k_m = m/a$, where a is the minor radius of plasma; $\eta(t)$ can simulate random phase fluctuations. Under the electrostatic field mentioned above can cause the occurrence the electrostatic drift islands in the drift surface family. In this paper it is derived the analytical equation of the drift surface to describe the electrostatic drift islands and the transfer the resonant (electrostatic resonant) passing particles into the trapped particles. With the numerical study it is also shown, that in helical plasma passing impurity particle becomes trapped under the drift-wave-like electric field. Similarly to the estafette (relay race) of drift resonances [1], the impurity ion feels the drift wave electric field sequentially and moves outside of the confinement volume and escapes.

The aim of this paper is to consider the effect of two chains of electrostatic islands on the particle escape from the confinement volume periphery and to analyze the possibility of the heavy impurity removal from the helical plasma periphery with the help of drift-wave-like electric field.

ANALYTICAL STUDY

First of all we analyze the description of the particle motion in the guiding center approximation [2, 3]. For this purpose as it known we should solve the equation

$$\mathbf{B}^*\nabla\Psi^* = 0, \tag{1}$$

where $\mathbf{B}^*$ is the so called effective magnetic field and $\Psi^* = const$ is the equation of drift surface of the charge particle [3].

The effective magnetic field can be presented as the sum of contributions proportional to magnetic field itself and the drift terms in the guiding center equations

$$\mathbf{B}^* = \mathbf{B} + \mathbf{B}^*_{[\mathbf{B}\times\nabla B]} + \mathbf{B}^*_{[\mathbf{E}\times\mathbf{B}]}. \tag{2}$$

It means that the function Ψ^* also is the sum of the corresponding contributions

$$\Psi^* = \Psi_{\mathbf{B}} + \Psi^*_{[\mathbf{B}\times\nabla B]} + \Psi^*_{[\mathbf{E}\times\mathbf{B}]}. \tag{3}$$

In the case of the simple magnetic field model

$$\mathbf{B} = B_0\left(1-\frac{r}{R}\cos\vartheta\right)^{-1}\left\{0, \frac{r}{R}\iota(r^2), 1\right\} \tag{4}$$

and electric field potential in the form [4]

$$\widetilde{\Phi} = \widetilde{\Phi}_{m,n,0}\, e^{-\frac{\sigma_I}{2}(r-r_0)^2}\cos\left[-\frac{\sigma_R}{2}(r-r_0)^2 - m\vartheta + n\varphi - \omega^* t\right] \tag{5}$$

we obtain the following analytical expressions for the contributions in the expression for Ψ^*

$$\Psi_{\mathbf{B}} = \int\left[m\iota(r^2) - n\right]r dr\,, \tag{6.1}$$

$$\Psi^*_{[\mathbf{B}\times\nabla B]} = (-1)\frac{1}{\iota(r^2)}\frac{Mc}{2eB_0}\frac{2V_\parallel^2 + V_\perp^2}{V_\parallel}\left[m\iota(r^2) - n\right]r\cos\vartheta\,, \tag{6.2}$$

$$\Psi^*_{[\mathbf{E}\times\mathbf{B}]} = \frac{c}{V_\parallel}\frac{R\widetilde{\Phi}_{m,n,0}}{B_0}(-1)e^{-\frac{\sigma_I}{2}(r-r_0)^2}\left[m + n\iota\left(\frac{r}{R}\right)^2\right]\cos\left[-\frac{\sigma_R}{2}(r-r_0)^2 - m\vartheta + n\varphi - \omega^* t\right]. \tag{6.3}$$

The procedure to obtain the last expressions is similar to the procedure used in [1] to obtain the drift surfaces in the case of the magnetic field resonances. The expression (3) for Ψ^* with (6.1)-(6.3) describes the electrostatic drift island structure with the half width in the form

$$\Delta r_{m,n,\widetilde{\Phi}} \propto \sqrt{\left(\frac{\widetilde{\Phi}_{m,n,0}}{aB_0}\right)\frac{rR}{d\iota/d(r/a)^2}}\,. \tag{7}$$

The size of islands depends on the square root of electrostatic field to magnetic field ratio and the shear of the magnetic field. In all realistic cases there are present the set of harmonics $\widetilde{\Phi} = \sum_{m,n}\widetilde{\Phi}_{m,n,0}\cos\left[m\vartheta + n\varphi - \delta_{m,n}\right]$ with different values of m,n and on adjacent resonant surfaces. It means that some resonances can take place with drift islands and their cross sizes are $\Delta r_{m,n,\widetilde{\Phi}}$, $\Delta r_{m',n',\widetilde{\Phi}}$, $\Delta r_{m'',n'',\widetilde{\Phi}}$ and so on. If there exist such conditions that

$$K_\Phi \equiv \frac{\Delta_{m,n,\widetilde{\Phi}} + \Delta_{m',n',\widetilde{\Phi}} + \Delta_{m'',n'',\widetilde{\Phi}}}{r_{m'',n''} - r_{m,n}} \geq 1 \tag{8}$$

we can expect the overlapping of the resonant structures and particle enhanced transport across magnetic field.

MODELS AND BASIC EQUATIONS OF THE NUMERICAL STUDY

We would like to demonstrate the escape of the impurity ion under the electric drift-wave-like field in the helical magnetic field of the torsatron type magnetic device Uragan-3M with the numerical study.

Confining magnetic field

The magnetic field model simulates a set of closed magnetic surfaces. The magnetic surfaces cross-sections are shown on the Fig. 1.The confining magnetic field in the helical magnetic trap is simulated by magnetic field potential

$$\Phi_M = B_0\left(R\varphi - \frac{R}{m}\sum_n \varepsilon_{n,m}\left(\frac{r}{a}\right)^n\sin(n\vartheta - m\varphi) + \varepsilon_{1,0} r\sin\vartheta\right). \tag{9}$$

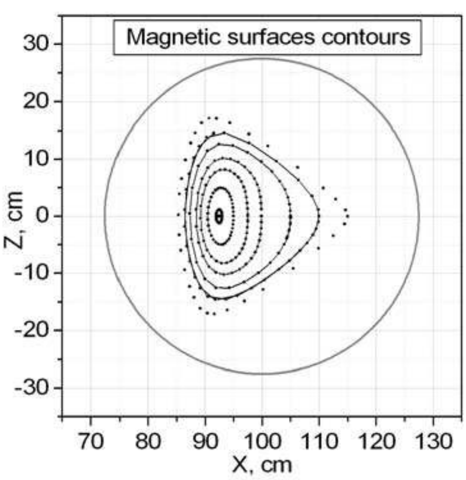


Fig. 1. Magnetic surfaces cross-sections.

Here r, ϑ, φ are the radial, poloidal and toroidal coordinates connected correspondingly, B_0 is the magnetic field magnitude at the magnetic axis, R and a are the major and minor radii of the torus, $n = l$ is the number of helical winding poles (number of the magnetic field periods along the minor around of torus), m is the number of magnetic field periods along the major around of torus, $\varepsilon_{n,m}$ are the coefficients of appropriate harmonics of the magnetic field. Magnetic field is derived from this expression as $\mathbf{B} = \nabla\Phi_M$. For our consideration we take the parameters of the torsatron Uragan-3M, namely, B_0 =1.2 T, R =100 cm, a =27.5 cm, l =3, m =9, $\varepsilon_{1,0}$ =0.275, $\varepsilon_{3,9}$ =0.65, $\varepsilon_{4,9}$ =0.032, $\varepsilon_{2,9}$ =-0.056. The magnetic field absolute value is given by $B \approx B_\varphi$.

Electric field

For the electric field which accompanies the drift-wave-like processes there are used some models [4, 5]. In our numerical calculations we use the following drift wave potential expression [5]:

$$\tilde{\Phi}_E(r,\vartheta,\varphi) = \tilde{\Phi}_{E0} e^{-\sigma_I (r-r_0)^2/2} \cos\left[-\sigma_R (r-r_0)^2/2 + l_E\vartheta + m_E\varphi\right]. \quad (10)$$

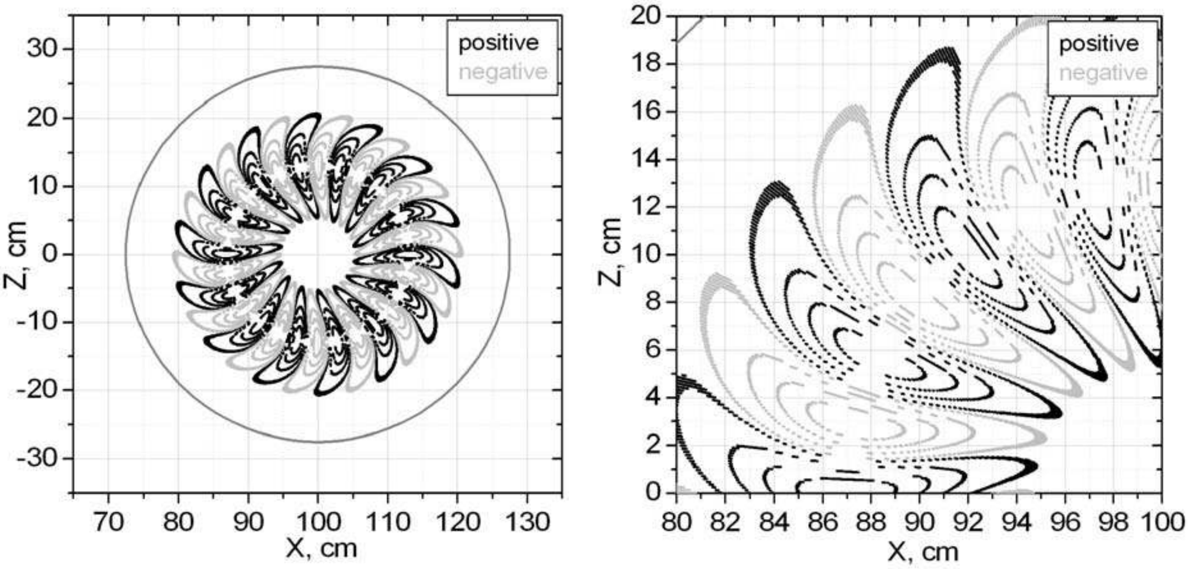
The equipotential lines on the vertical cross-section of the confinement volume are shown on the Fig. 2. As it is seen on the Fig. 2a, the electric field potential has some interchangeable areas of different potential value. Here we take the drift-wave-like potential with $l_E = 14$ and $m_E = 11$, $\sigma_R a^2 = \sigma_I a^2 = 0.1$, r_0 =16 cm. Potential peaks are placed on a circle with the radius of r_0 on the vertical plane. Black lines correspond to the positive potential and grey lines correspond to the negative potential. The enlarged structure of the electric field potential within ¼ of the minor round of torus (in the poloidal direction) is shown on the Fig. 2b.

The particle motion equation

The charged particle motion in the electromagnetic field is studied by the numerical integration of the Newton-Lorentz (gyro-orbit) equation:

$$M\frac{d\mathbf{v}}{dt} = Ze\mathbf{E} + \frac{Ze}{c}[\mathbf{v}\times\mathbf{B}], \quad (11)$$

where M - mass, $\mathbf{v}$ - velocity, Z - charge number of the test particle. The tungsten ion with Z =30 and energy 1 keV is taken as the test particle.



A b

Fig. 2. Vertical cross-section of the electric field potential (equipotential lines), a – general view, b – enlarged section.

RESULTS OF THE NUMERICAL STUDY
Two drift-wave-like potentials of the type (10)

The drift wave potential is simulated to have two items:

$$\Phi_E(r-r_{01}, r-r_{02}) = \Phi_{E1}(r-r_{01}) + \Phi_{E2}(r-r_{02}). \quad (12)$$

Thus the radial distribution of the electric field has two "peaks", as it is shown on the Fig. 3, at the radius r_{01}=13 cm and r_{02}=16 cm with the relative amplitudes of $\Phi_{E01}/aB_0 = 2\times10^{-7}$ and $\Phi_{E02}/aB_0 = 3\times10^{-7}$. The estimated values of the electric field magnitude are about 3 V/cm for the inner chain of electrostatic islands and 4.5 V/cm for the outer chain. The test particle moves in the confinement volume under electric field. The spatial structure of the electric field with two peaks is seen on the Fig. 4, where the equipotential lines of the electric field are presented. Lines of positive potential are in black, negative – in grey. The start position and the initial pitch of the test particle are chosen near the boundary for the test particle to be passing without the influence of the drift wave electric field. It means that if we take some smaller value of the initial pitch or some higher value of the start point radial coordinate, then the test particle will be initially trapped. The passing test particle orbit is presented on the Fig. 5. The line thickness on the figure corresponds to doubled Larmour radius, which is 3 mm.

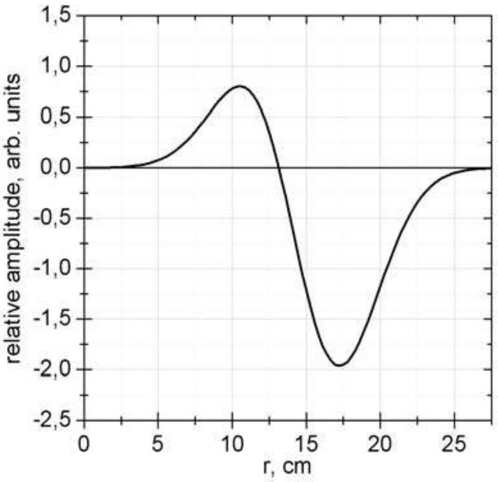


Fig. 3. Radial distribution of the electric field potential

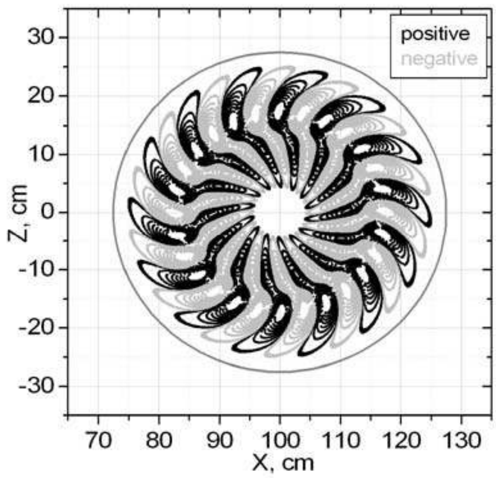


Fig. 4. Equipotential lines of the electric field.

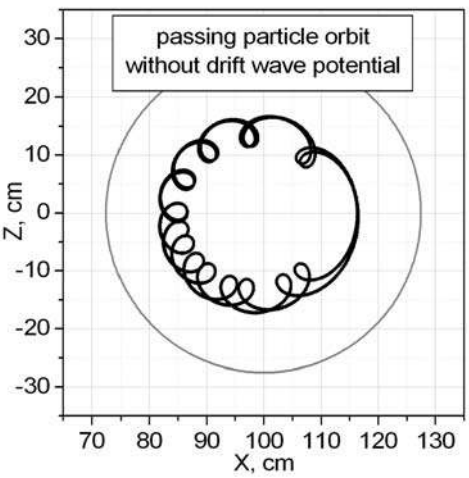


Fig. 5. Passing test particle orbit.

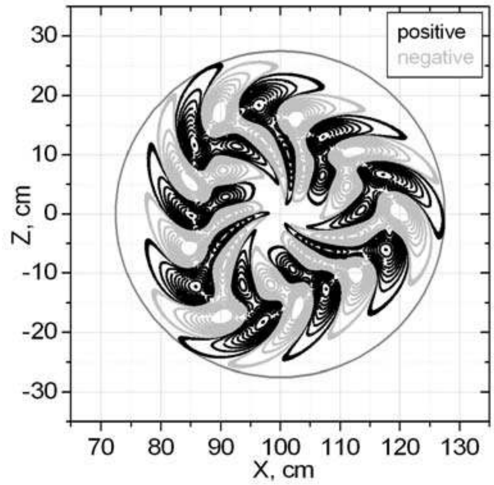


Fig. 6. Radial distribution of the electric field potential of the improved model

Particle motion under two drift-wave-like potentials changed in correspondence with the shape of the magnetic surfaces

In this section we assume electric field potential dependence on coordinates and do not take into account its dependence on time i.e. ω^*=0, see expression (5). The model of the electric field potential is improved to correspond to the shape of the magnetic surface cross-sections with $l_E = 3$ and $m_E = 9$. The expression for the electric field potential is as follows:

$$\begin{aligned}\widetilde{\Phi}_E(r,\vartheta,\varphi) = \widetilde{\Phi}_{E01}\exp\left\{-\sigma_I\left(r - r_{01}(\alpha_1 + \beta_1\cos l_E\vartheta)\right)^2/2\right\}\cos\left[-\sigma_R\left(r - r_{01}(\alpha_1 + \beta_1\cos l_E\vartheta)\right)^2/2 + l_E\vartheta + m_E\varphi\right] + \\ + \widetilde{\Phi}_{E02}\exp\left\{-\sigma_I\left(r - r_{02}(\alpha_2 + \beta_2\cos l_E\vartheta)\right)^2/2\right\}\cos\left[-\sigma_R\left(r - r_{02}(\alpha_2 + \beta_2\cos l_E\vartheta)\right)^2/2 + l_E\vartheta + m_E\varphi\right].\end{aligned} \quad (13)$$

Here coefficients $\alpha_1 = \alpha_2$=1, β_1=5/48, β_2=5/32; they are responsible for the triangular shape of the equipotential line contours seen on the Fig. 6. In other aspects this expression is similar to (10) and (12).

These changes concern the layout of the electric field potential peaks on the vertical cross-section of the confinement volume. Similarly to Fig. 4, the equipotential lines of the improved electric field model are presented on the Fig. 6. One can see triangular shape, which coincides with the magnetic surface shape. It is necessary to point out that the effect of these two potentials taken separately does not lead to particle escape, but only to its transition into trapped state.

When we assume Φ_{E02}=0 in expression (13), we have only inner chain of the electrostatic potential cells at the average radius of 13 cm. The test particle becomes trapped for a short time, but stays in the confinement volume. When we leave only outer chain of the electrostatic potential cells at the average radius of 16 cm (by assuming Φ_{E01}=0), the effect of the electric field on the test particle is poor, because it moves at a distance from the region, where the electric field takes remarkable values. However the test particle becomes trapped, but does not escape from the confinement volume. Particle orbits, corresponding to these cases, are presented on the Fig. 7 and Fig. 8.

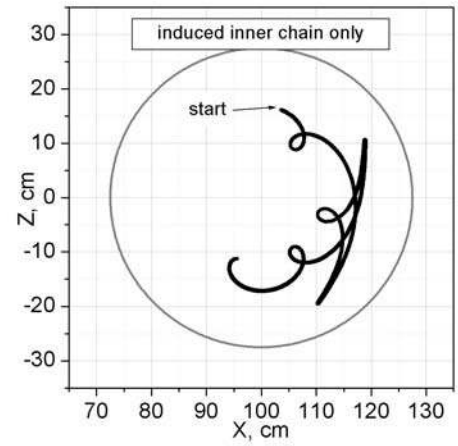


Fig. 7. Test particle orbit under the potential $\Phi_1(r - r_{01})$ taken separately.

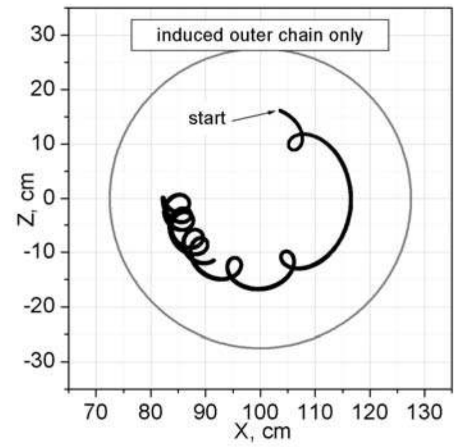


Fig. 8. Test particle orbit under the potential $\Phi_2(r - r_{02})$ taken separately.

Particle escape under two drift-wave-like potentials

If the radial coordinate (position) of the test particle is close to r_{01} then it feels the effect of the electric field potential $\Phi_{E1}(r - r_{01})$ and transits into the trapped state. As the test particle is trapped, it deviates from the initial magnetic surface on a remarkable distance and comes under the influence of the electric field potential $\Phi_{E2}(r - r_{02})$. The effect of this potential on the test particle is that the test particle escapes the confinement volume. The vertical cross-section of the escaping particle orbit is shown on the Fig. 9. The top view of the particle orbits is presented on the Fig. 10. Grey line corresponds to the passing particle orbit, which corresponds to Fig. 5, black line indicates the orbit of the escaping test particle under the drift-wave electric field.

Although the backward impurity transition from the trapped state into the passing one is possible, the penetration of the impurity into the core plasma is not expected.

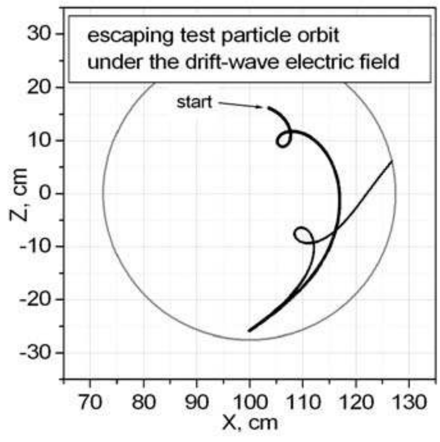


Fig. 9. Effect of two drift-wave potentials on the test particle orbit.

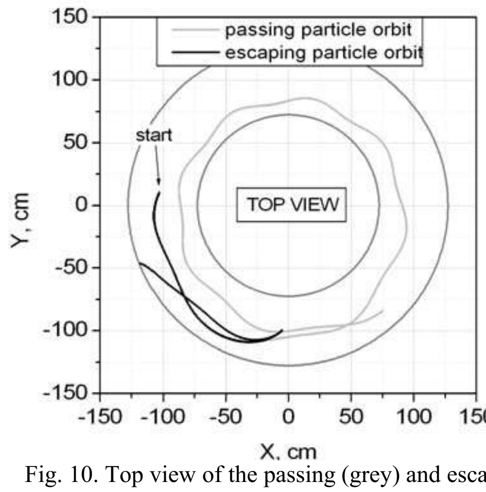


Fig. 10. Top view of the passing (grey) and escaping (black) test particle orbits.

Particle motion under two drift-wave-like potentials and nonzero drift wave frequency

In this section we take into account electric field potential dependence either on coordinates or on time, i.e. $\omega^* \neq 0$, namely $\omega^* = 1.71 \cdot 10^7$ sec^{-1} for the inner chain of electrostatic islands and $\omega^* = 1.61 \cdot 10^7$ sec^{-1} for the outer chain. These values are taken using the expression for ω^* in [7]. The same test particle has the same start position and its orbit in absence of the electric field is presented on Fig. 5. Similarly to the case of ω^*=0, each chain of the electrostatic islands taken separately does not cause particle escape from the magnetic volume, but causes particle transition into the trapped state, as it is shown on Fig. 11 and Fig. 12.

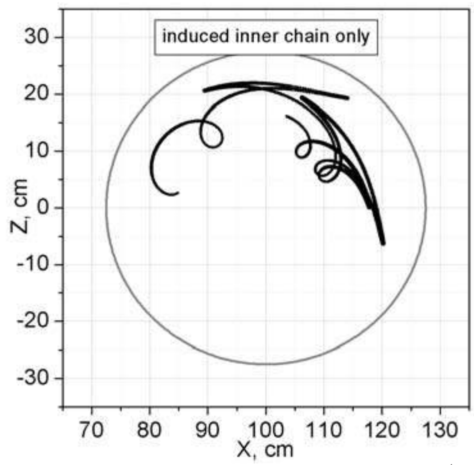


Fig. 11. Test particle orbit under the potential $\Phi_1(r - r_{01})$ taken separately. $\omega^* = 1.71 \cdot 10^7$ sec^{-1}.

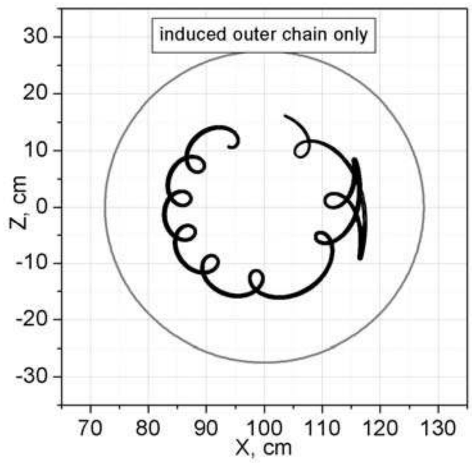


Fig. 12. Test particle orbit under the potential $\Phi_2(r - r_{02})$ taken separately. $\omega^* = 1.61 \cdot 10^7$ sec^{-1}.

The spatial distribution of the electric field potential at the fixed moment of time at the certain vertical cross-section of the torus is the same as that shown on the Fig. 6. When the test particle moves under the electric field of two chains of electrostatic islands it becomes trapped and then escape from the confinement volume. The corresponding orbit of the escaping test particle is presented on the Fig. 13. The enlarged plot of the reflection point of the escaping particle orbit is shown on the Fig. 14. The Larmour radius of the test particle can be easily estimated.

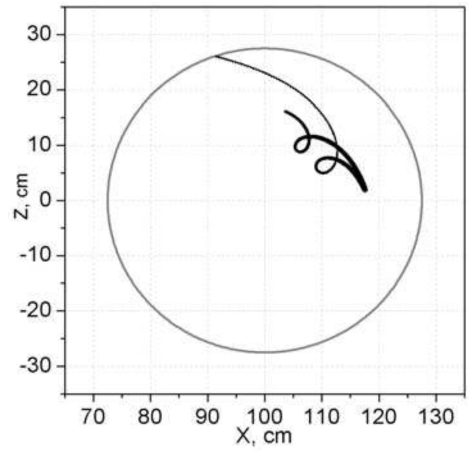


Fig. 13.Escaping test particle orbit. $\omega^* \neq 0$. Both chains are induced.

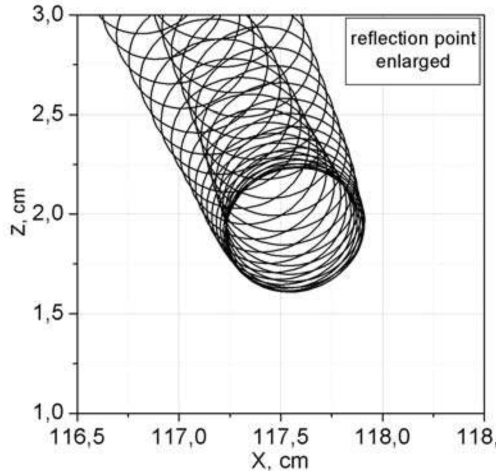


Fig. 14. Enlarged reflection point of the test particle orbit shown on Fig. 13. $\omega^* \neq 0$. Both chains are induced.

DISCUSSION

We would like to discuss some possible critical issues of the carried out study.

1. Backward impurity transition from the trapped state into the passing one is possible however the penetration of the impurity into the core plasma is not expected. To be sure that it is so, the statistics of impurity transport is desirable. It will be done in the future study.
2. In this paper we use the electric field potential which was derived in paper [7] for the tokamak like magnetic configuration. We only modify that expression to satisfy the physics picture corresponding to $l = 3$ torsatron like configuration. In the general consideration in [7] there is also the dependence of $\widetilde{\Phi}_E$ on $\omega^* t$, where ω^* is the drift frequency. We have carried out study in the approximation when $\omega^* \to 0$ and in the case of $\omega^* \neq 0$ and it is different for each electrostatic island radial position. The expression for the drift frequency ω^* can be found in [7].

CONCLUSION

In this paper it is shown that a sequence of two chains of electrostatic islands can be the reason of particle escape from the confinement volume periphery. It is important to point out that a single chain of electrostatic islands can cause particle transport in both directions, either outside of the confinement volume or from the periphery into the core plasma. Two consequently displaced layers of electrostatic field should decrease particle transport towards the plasma core. It is also shown that the in the case of non zero drift wave frequency (in the case of drift waves existence) the process of removal take place due to resonant particle - electric field interaction

We would like to stress on the possibility of the heavy impurity removal from the helical plasma periphery with the help of drift-wave-like electric field. The impurity ion transits from passing state into the trapped one under the effect of the electric field and escapes from the magnetic volume due to the natural drift in the inhomogeneous magnetic field. With regard to drift wave frequency range, the described effect mainly concern and directed to heavy impurity ion transport in plasma periphery.

This effect can be observed in torsatron configurations and particularly on Uragan-3M device.

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ПЕРЕНОС ПРИМЕСНЫХ ИОНОВ ПОД ВОЗДЕЙСТВИЕМ ЭЛЕКТРОСТАТИЧЕСКОГО ПОЛЯ ДРЕЙФОВОЙ ВОЛНЫ

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Перенос тяжелых ионов примеси рассмотрен вблизи смежных рациональных магнитных поверхностей при наличии электрического потенциала дрейфовой волны. Аналитическое исследование проделано для упрощенной модели магнитного поля в тороидальной магнитной ловушке с вращательным преобразованием магнитных силовых линий. Исследование с помощью численных методов проведено для винтовой плазмы. Орбиты пробных частиц построены путем численного интегрирования системы уравнений Ньютона для заряженной частицы в магнитном поле и квазистатическом электрическом поле. В качестве пробной частицы взят ион вольфрама.

КЛЮЧЕВЫЕ СЛОВА: электрическое поле дрейфовой волны, электростатические острова, переход частицы в запертое состояние, перенос примесного иона, винтовое магнитное поле