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VLASOV EQUATION FOR MAGNETIZED PLASMA PARTICLES IN THE ARBITRARY MAGNETIC FIELD

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The linearized Vlasov equation is rewritten for charged particles in the two-dimensional axisymmetric plasma models using the cylindrical coordinates. There is described a method of its solution by the Fourier expansions of the perturbed distribution functions over the gyrophase angle in velocity space, conservation integrals of particle motion in the curvilinear magnetic field and smallness of the magnetization parameters. Such an approach allows us to evaluate the main contributions of untrapped and trapped particles to the transverse and longitudinal dielectric tensor components for electromagnetic waves in tokamaks, straight mirror-traps, laboratoty dipole magnetospheric plasmas and inner part of the Earth's magnetosphere.
KEY WORDS: Vlasov equation, kinetic wave theory, tokamaks, mirror traps, magnitospheric plasmas.

УРАВНЕНИЕ ВЛАСОВА ДЛЯ ЗАМАГНИЧЕННЫХ ЧАСТИЦ ПЛАЗМЫ В ПРОИЗВОЛЬНОМ МАГНИТНОМ ПОЛЕ

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Представлено линеаризованное уравнение Власова в цилиндрических координатах для заряженных частиц в двумерно-неоднородных аксиально-симметричных моделях плазмы. Описан метод его решения для пролетных и запертых частиц используя Фурье-разложение возмущенных функций распределения частиц по углу циклотронного вращения в пространстве скоростей, инварианты движения частиц в криволинейном магнитном поле и малость параметра замагниченности плазмы. Такой подход позволяет вычислить основной вклад пролетных и запертых частиц в поперечные и продольную компоненты тензора диэлектрической проницаемости электромагнитных волн в токамаках, цилиндрических пробкотронах, лабораторной диполярной магнитосферной плазме, околоземной магнитосфере.
КЛЮЧЕВЫЕ СЛОВА: уравнение Власова, кинетическая теория волн, токамак, цилиндрический пробкотрон, магнитосферная плазма.

РІВНЯННЯ ВЛАСОВА ДЛЯ ЗАМАГНІЧЕНИХ ЧАСТИНОК ПЛАЗМИ У ДОВІЛЬНОМУ МАГНИТНОМУ ПОЛІ

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Наведено лінеаризоване рівняння Власова в циліндричних координатах для заряджених частинок в двовимірно-неоднорідних аксіально-симетричних моделях плазми. Описано метод його розв'язку для пролітних і запертих частинок використаючи Фур'е-розклад збуджених функцій розподілу частинок по куту циклотронного обертання у просторі швидкостей, інваріанти руху частинок в криволінійному магнітному полі та малість параметру замагнічності плазми. Такий підхід дозволяє розрахувати основний внесок пролітних і запертих частинок в поперечні і подовжню компоненти тензора діелектричної проникності для електромагнітних хвиль в токамаках, циліндричних пробкотронах, лабораторній диполярній магнітосферній плазмі, навколоземній магнітосфері.
КЛЮЧОВІ СЛОВА: рівняння Власова, кінетична теорія хвиль, токамак, циліндричний пробкотрон, магнітосферна плазма.

Since plasma is an ensemble of charged particles (ions and electrons) its behavior can be described by the kinetic equation for probability distribution functions, $F_\alpha(t,\mathbf{r},\mathbf{v})$, of α-kind particles in the six-dimensional phase $(\mathbf{r},\mathbf{v})$-volume. In the general case, $F_\alpha(t,\mathbf{r},\mathbf{v})$ is a function of seven variables: *t*-time, three variables in velocity space **v**, and three variables in geometric space **r**. In plasma theory, the corresponding kinetic equation is known as the Vlasov equation [1] or collisionless Boltzmann equation, where the generalized force acting the particles is defined as the self-consistent Lorentz force. As a result, the linearized Vlasov equation for the perturbed distribution functions $f_\alpha(t,\mathbf{r},\mathbf{v}) = F_\alpha(t,\mathbf{r},\mathbf{v})\text{-}F_{0\alpha}(\mathbf{r},\mathbf{v})$ can be written as

$$\frac{\partial f_\alpha}{\partial t} + (\mathbf{v}\nabla) f_\alpha + \frac{e_\alpha H_0}{M_\alpha c}[\mathbf{v}\times\mathbf{h}]\frac{\partial f_\alpha}{\partial \mathbf{v}} = -\frac{e_\alpha}{M_\alpha}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{H}}{c}\right)\frac{\partial F_{0\alpha}}{\partial \mathbf{v}}. \quad (1)$$

Here $F_{0\alpha}(\mathbf{r},\mathbf{v})$ is the steady-state (as well as equilibrium and non-equilibrium) distribution functions of particles with the mass M_α and charge e_α; **E** and **H** are the perturbed electric and magnetic fields; H_0 is the modulus of an equilibrium magnetic field $\mathbf{H}_0$; $\mathbf{h} = \mathbf{H}_0 / H_0$; *c* is the speed of light; $\nabla = \partial / \partial \mathbf{r}$.

After solving Eq. (1), one can calculate the basic moments of plasma distribution functions such as the perturbation of plasma density $n(t,\mathbf{r})$ (as the zeroth moment of f_α):

$$n(t,\mathbf{r}) = \sum_{\alpha}^{e,i_1,i_2,...} \int_{\mathbf{v}} f_\alpha(t,\mathbf{r},\mathbf{v})d\mathbf{v}, \tag{2}$$

the perturbed current density components, $\mathbf{j}(t,\mathbf{r})$ (as the first moments of f_α):

$$\mathbf{j}(t,\mathbf{r}) = \sum_{\alpha}^{e,i_1,i_2,...} e_\alpha \int_{\mathbf{v}} \mathbf{v} f_\alpha(t,\mathbf{r},\mathbf{v})d\mathbf{v}, \tag{3}$$

the plasma pressure transverse and along the $\mathbf{H}_0$-field lines (as the second moments of f_α), heat conductivity components and others.

It is well known, any wave process in magnetized plasmas can be described by solving the Maxwell's equations for the perturbed (**E**,**H**)-components:

$$[\nabla\times\mathbf{E}] = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \qquad [\nabla\times\mathbf{H}] = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j}, \tag{4}$$

where the Gaussian system of units has been used. The set of Eqs. (4) will be complete if we know the relation between **j** and (**E**,**H**)-fields. Usually this connection, for the harmonic oscillations proportional to $\exp(-\mathrm{i}\omega t)$, is defined by the wave conductivity tensor σ_{ik}: $j_i = \sigma_{ik}E_k$, or by the dielectric tensor ε_{ik}: $\varepsilon_{ik} = \delta_{ik} + 4\pi\mathrm{i}\sigma_{ik}/\omega$, where δ_{ik} are the Kronecker constants; i,k=1,2,3 indicate the vector projections, ω is the wave frequency. It means that before solving Eqs. (4) we should calculate the ε_{ik} (or σ_{ik}) tensor components by solving Eq. (1) and using Eq. (3).

The main feature of magnetized plasmas is the fact that their dielectric tensor components have the different form for different plasma models. This form depends substantially on the wave frequency ω, the plasma parameters (density N, temperature T) and the geometry of an equilibrium magnetic field $\mathbf{H}_0(\mathbf{r})$. Presently, the linear wave theory is developed very well for the plane waves in both the isotropic (when $\mathbf{H}_0$=0) and anisotropic magnetized plasmas in the straight magnetic field, see, e.g., Ref. [2] and the bibliography therein.

However, the approximation of plane waves is not suitable for such realistic plasma systems as the inner part of the Earth's magnetosphere, laboratory dipole magnetosphere, straight mirror traps and tokamaks. All these plasmas can be modeled as two-dimensional (2D) axisymmetric configurations with one minimum of a nonuniform equilibrium magnetic field; where plasma particles should be split in the two populations of the so-called trapped and untrapped (or passing, or circulating) particles. Accordingly, Eq. (1) can be resolved separately as a boundary value problem for each particle group.

The main aim of this paper is to derive the kinetic equation in the convenient form for 2D magnetospheric, toroidal and mirror-trapped plasma models in the collisionless limit. Of course, the initial Eq. (1) should be resolved separately for each specific 2D plasma configuration using one set of coordinates or another. However, since the above mentioned models are axisymmetric it is convenient to use the usual cylindrical coordinates for plasmas in the arbitrary three-dimensional $\mathbf{H}_0$-field.

THE LINEARIZED VLASOV EQUATION

Describing the axisymmetric plasma configurations (such as tokamaks, Earth's radiation belts, laboratory dipole magnetosphere, straight mirror traps etc) it is convenient to use the cylindrical coordinates (ρ, ϕ, z), where the axis z is coinciding with the main symmetry axis. In this case, the linearized Vlasov equation for perturbed distribution functions, $f_\alpha(t,\mathbf{r},\mathbf{v}) = f_\alpha(t,\rho,\phi,z,v_\parallel,v_\perp,\sigma)$, of charged particles (ions and electrons, $\alpha = e,i_1,i_2,...$) can be rewritten as

$$\frac{\partial f}{\partial t} + v_\parallel(\mathbf{h}\nabla)f + \frac{v_\perp}{2}(\nabla\mathbf{h})\hat{V}f - \left\{\Omega_c + \frac{v_\parallel}{2}\Big[\mathbf{2b(h\nabla)n} + \mathbf{h(b\nabla)n} - \mathbf{h(n\nabla)b} + \right.$$
$$\left. +2\frac{h_\phi h_z}{\rho} - \frac{b_\phi b_z}{\rho} - \frac{n_\phi n_z}{\rho}\Big]\right\}\frac{\partial f}{\partial\sigma} + \cos\sigma\left\{v_\perp(\mathbf{n}\nabla)f + v_\parallel\left[\mathbf{n(h\nabla)h} + \frac{h_\phi b_z}{\rho}\right]\hat{V}f + \right.$$
$$\left. + \frac{1}{v_\perp}\left[v_\perp^2\mathbf{n(n\nabla)b} + v_\parallel^2\mathbf{h(h\nabla)b} + v_\parallel^2\frac{h_\phi n_z}{\rho} - v_\perp^2\frac{h_z n_\phi}{\rho}\right]\frac{\partial f}{\partial\sigma}\right\} + \sin\sigma\left\{v_\perp(\mathbf{b}\nabla)f + \right.$$
$$\left. + v_\parallel\left[\mathbf{b(h\nabla)h} - \frac{h_\phi n_z}{\rho}\right]\hat{V}f - \frac{1}{v_\perp}\left[v_\perp^2\mathbf{b(b\nabla)n} + v_\parallel^2\mathbf{h(h\nabla)n} - v_\parallel^2\frac{h_\phi b_z}{\rho} + v_\perp^2\frac{h_z b_\phi}{\rho}\right]\frac{\partial f}{\partial\sigma}\right\} +$$

$$+\frac{\cos 2\sigma}{2}\left\{v_\perp\left[\mathbf{n}(\mathbf{n}\nabla)\mathbf{h}-\mathbf{b}(\mathbf{b}\nabla)\mathbf{h}+\frac{n_\phi b_z}{\rho}+\frac{b_\phi n_z}{\rho}\right]\hat{V}f+v_\parallel\left[\mathbf{h}(\mathbf{b}\nabla)\mathbf{n}-\frac{b_\phi b_z}{\rho}+\right.\right.$$
$$\left.\left.+\mathbf{h}(\mathbf{n}\nabla)\mathbf{b}+\frac{n_\phi n_z}{\rho}\right]\frac{\partial f}{\partial\sigma}\right\}+\frac{\sin 2\sigma}{2}\left\{v_\perp\left[\mathbf{n}(\mathbf{b}\nabla)\mathbf{h}+\mathbf{b}(\mathbf{n}\nabla)\mathbf{h}+\frac{b_\phi b_z}{\rho}-\frac{n_\phi n_z}{\rho}\right]\hat{V}f-\right. \quad (5)$$
$$\left.-v_\parallel\left[\mathbf{h}(\mathbf{n}\nabla)\mathbf{n}-\mathbf{h}(\mathbf{b}\nabla)\mathbf{b}-\frac{n_\phi b_z}{\rho}-\frac{b_\phi n_z}{\rho}\right]\frac{\partial f}{\partial\sigma}\right\}=$$
$$=-\frac{e}{M}\left\{E_\parallel\frac{\partial F_0}{\partial v_\parallel}+\cos\sigma\left[E_n\frac{\partial F_0}{\partial v_\perp}+\frac{H_b}{c}\hat{V}F_0+\frac{1}{v_\perp}\left(E_b+\frac{v_\parallel}{c}H_n\right)\frac{\partial F_0}{\partial\sigma}\right]-\right.$$
$$\left.-\frac{H_\parallel}{c}\frac{\partial F_0}{\partial\sigma}+\sin\sigma\left[E_b\frac{\partial F_0}{\partial v_\perp}-\frac{H_n}{c}\hat{V}F_0-\frac{1}{v_\perp}\left(E_n-\frac{v_\parallel}{c}H_b\right)\frac{\partial F_0}{\partial\sigma}\right]\right\}.$$

where F_0 is the equilibrium (or steady-state) distribution function. The index α of the particle species is omitted, and the additional definitions are

$$\hat{V}f=v_\perp\frac{\partial f}{\partial v_\parallel}-v_\parallel\frac{\partial f}{\partial v_\perp}, \qquad \Omega_c=\frac{eH_0}{Mc}. \quad (6)$$

In Eq. (5), for the vector values $\mathbf{A}=\{\mathbf{E},\mathbf{H},\mathbf{v},\mathbf{j}\}$, we use the normal A_1, binormal A_2, and parallel A_3 projections relative to an equilibrium (three-dimensional in the general case) magnetic field $\mathbf{H}_0(\mathbf{r})$:

$$\mathbf{A}=A_1\mathbf{n}+A_2\mathbf{b}+A_3\mathbf{h}, \quad (7)$$

so that

$$A_1\equiv A_n=A_\rho n_\rho+A_\phi n_\phi+A_z n_z, \quad A_2\equiv A_b=A_\rho b_\rho+A_\phi b_\phi+A_z b_z, \quad A_3\equiv A_\parallel=A_\rho h_\rho+A_\phi h_\phi+A_z h_z, \quad (8)$$

where **n**, **b**, **h** are the normal, binormal and parallel unit vectors relative to $\mathbf{H}_0$:

$$\mathbf{h}=\mathbf{H}_0/H_0, \qquad \mathbf{n}=\mathbf{b}\times\mathbf{h}, \qquad \mathbf{b}=\mathbf{h}\times\mathbf{n}. \quad (9)$$

Moreover, in velocity space we use the polar coordinates ($v_\perp,\sigma$) instead of (v_1,v_2) by the transformation

$$v_1=v_\perp\cos\sigma, \qquad v_2=v_\perp\sin\sigma, \qquad v_3=v_\parallel. \quad (10)$$

AXISYMMETRIC TWO-DIMENSIONAL TOKAMAKS

For axisymmetric tokamaks, Eq. (5) can be readily simplified under the conditions i) $\mathbf{H}_0$ is independent of ϕ, and ii) the normal component (perpendicular to the magnetic surface) of an equilibrium magnetic field is equal to zero, i.e., when $\mathbf{n}\cdot\mathbf{H}_0=0$. The corresponding kinetic equations, their solutions and dielectric tensor evaluation for radio-frequency waves in the toroidal plasmas with an arbitrary tokamak aspect ratio have been present in Refs. [3-5], respectively, for circular, elliptic and D-shaped magnetic surfaces.

In particular, for axisymmetric tokamaks with circular magnetic surfaces, the cylindrical projections of an equilibrium magnetic field (in the case when the Shafranov shift of the magnetic surfaces is neglected) have the forms:

$$H_{0r}=0, \qquad H_{0\theta}=\frac{\bar{H}_{0\theta}(r)}{1+\varepsilon\cos\theta}, \qquad H_{0\phi}=\frac{\bar{H}_{0\phi}(r)}{1+\varepsilon\cos\theta}. \quad (11)$$

Here $\varepsilon=r/R_0$ is the inverse tokamak aspect ratio; and quasi-toroidal coordinates (r,θ,ϕ) have been used instead of the cylindrical ones (ρ,ϕ,z) as

$$\rho=R_0+r\cos\theta, \qquad \phi=\phi, \qquad z=-r\sin\theta, \quad (12)$$

where R_0 is the major radius of plasma torus; r is the radius of the considered magnetic surface; θ and ϕ are the poloidal and toroidal angles, respectively; $\bar{H}_{0\theta}=H_{0\theta}(r,\pi/2)$ and $\bar{H}_{0\phi}=H_{0\phi}(r,\pi/2)$ are the poloidal and toroidal projections of an equilibrium magnetic field. After the such transformations, the linearized Vlasov equation (5) can be rewritten in the form

$$\frac{\partial f}{\partial t}-\Omega_c\frac{\partial f}{\partial\sigma}+v_\perp\cos\sigma\left[\frac{\partial f}{\partial r}+h_\phi^2\frac{\partial}{\partial r}\left(\frac{h_\theta}{h_\phi}\right)\left(\sin\sigma\hat{V}f-\frac{v_\parallel}{v_\perp}\cos\sigma\frac{\partial f}{\partial\sigma}\right)\right]+$$
$$+\frac{h_\theta v_\parallel+h_\phi v_\perp\sin\sigma}{r}\left(\frac{\partial f}{\partial\theta}-h_\theta\cos\sigma\hat{V}f-\frac{h_\phi v_\perp+h_\theta v_\parallel\cos\sigma}{v_\perp}\frac{\partial f}{\partial\sigma}\right)+\frac{h_\phi v_\parallel-h_\theta v_\perp\sin\sigma}{R_0+r\cos\theta}\times$$

$$\times\left[\frac{\partial f}{\partial\phi}-\cos\theta\left(h_\phi\cos\sigma\hat{V}f-\frac{h_\theta v_\perp-h_\phi v_\parallel\sin\sigma}{v_\perp}\frac{\partial f}{\partial\sigma}\right)+\sin\theta\left(\sin\sigma\hat{V}f-\frac{v_\parallel}{v_\perp}\cos\sigma\frac{\partial f}{\partial\sigma}\right)\right]=$$
$$=-\frac{e}{M}\left\{E_\parallel\frac{\partial F_0}{\partial v_\parallel}+\cos\sigma\left[E_n\frac{\partial F_0}{\partial v_\perp}+\frac{H_b}{c}\hat{V}F_0+\frac{1}{v_\perp}\left(E_b+\frac{v_\parallel}{c}H_n\right)\frac{\partial F_0}{\partial\sigma}\right]-\right.$$
$$\left.-\frac{H_\parallel}{c}\frac{\partial F_0}{\partial\sigma}+\sin\sigma\left[E_b\frac{\partial F_0}{\partial v_\perp}-\frac{H_n}{c}\hat{V}F_0-\frac{1}{v_\perp}\left(E_n-\frac{v_\parallel}{c}H_b\right)\frac{\partial F_0}{\partial\sigma}\right]\right\}. \tag{13}$$

Here the cyclotron frequency of plasma particles Ω_c is defined as usual by the modulus of $\mathbf{H}_0$:

$$\Omega_c=\frac{e}{Mc}\frac{\sqrt{\bar{H}_{0\phi}^2+\bar{H}_{0\theta}^2}}{1+\varepsilon\cos\theta}=\frac{\Omega_{c0}}{1+\varepsilon\cos\theta}. \tag{14}$$

MIRROR-TRAPPED PLASMAS

As regards to the magnitospheric plasma models, like as the inner part of the Earth's magnetosphere [6] and the Levitated Dipole eXperiment (LDX) plasma [7,8], and the straight mirror traps [9], Eq. (5) can be simplified substantially under the condition when the $H_{0\phi}$-component of $\mathbf{H}_0$ is equal to zero, $H_{0\phi}=0$. In this case, the vectors **n**, **b**, **h** have the following cylindrical projections: $\mathbf{h}=(h_\rho,0,h_z)$, $\mathbf{n}=(h_z,0,-h_\rho)$, $\mathbf{b}=(0,\ 1,\ 0)$, and Eq. (5) can be reduced to

$$\frac{\partial f}{\partial t}+v_\parallel h_\rho\frac{\partial f}{\partial\rho}+v_\parallel h_z\frac{\partial f}{\partial z}+\frac{v_\perp}{2}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho h_\rho+\frac{\partial h_z}{\partial z}\right)\hat{V}f-\Omega_c\frac{\partial f}{\partial\sigma}+$$
$$+\cos\sigma\left\{v_\perp h_z\frac{\partial f}{\partial\rho}-v_\perp h_\rho\frac{\partial f}{\partial z}-v_\parallel\left(\frac{\partial h_z}{\partial\rho}-\frac{\partial h_\rho}{\partial z}\right)\hat{V}f\right\}+$$
$$+\sin\sigma\left\{\frac{v_\perp}{\rho}\frac{\partial f}{\partial\phi}-\frac{1}{v_\perp}\left[v_\parallel^2\left(\frac{\partial h_z}{\partial\rho}-\frac{\partial h_\rho}{\partial z}\right)+v_\perp^2\frac{h_z}{\rho}\right]\frac{\partial f}{\partial\sigma}\right\}+$$
$$+\frac{v_\perp}{2}\cos2\sigma\left(\rho\frac{\partial}{\partial\rho}\frac{h_\rho}{\rho}+\frac{\partial h_z}{\partial z}\right)\hat{V}f+\frac{v_\parallel}{2}\sin2\sigma\left(\rho\frac{\partial}{\partial\rho}\frac{h_\rho}{\rho}+\frac{\partial h_z}{\partial z}\right)\frac{\partial f}{\partial\sigma}= \tag{15}$$
$$=-\frac{e}{M}\left\{E_\parallel\frac{\partial F_0}{\partial v_\parallel}+\cos\sigma\left[E_n\frac{\partial F_0}{\partial v_\perp}+\frac{H_b}{c}\hat{V}F_0+\frac{1}{v_\perp}\left(E_b+\frac{v_\parallel}{c}H_n\right)\frac{\partial F_0}{\partial\sigma}\right]-\right.$$
$$\left.-\frac{H_\parallel}{c}\frac{\partial F_0}{\partial\sigma}+\sin\sigma\left[E_b\frac{\partial F_0}{\partial v_\perp}-\frac{H_n}{c}\hat{V}F_0-\frac{1}{v_\perp}\left(E_n-\frac{v_\parallel}{c}H_b\right)\frac{\partial F_0}{\partial\sigma}\right]\right\}.$$

Further, the perturbed distribution function should be expanded in a Fourier series over the polar (or gyrophase) angle σ in velocity space,

$$f(t,\mathbf{r},\mathbf{v})=f(t,\rho,\phi,z,v_\parallel,v_\perp,\sigma)=\sum_l^{\pm\infty}f_l(\rho,z,v_\parallel,v_\perp)\exp(-i\omega t+in\phi-il\sigma), \tag{16}$$

accounting for that the problem is homogeneous in the time *t* and the angle ϕ; therefore the perturbed values (including the $\{\mathbf{E},\mathbf{H},\mathbf{j}\}$-components) are proportional to $\sim\exp(-i\omega t+in\phi)$, where *n* is the integer. Due to this procedure, we reduce the problem to solve the differential equations with respect to four partial derivatives for $f_l(\rho,z,v_\parallel,v_\perp)$-harmonics, whereas the initial equations (5), (13) and (15) were including the seven partial derivatives.

To evaluate the main contribution of plasma particles to the perturbed current density components it is enough to find the f_l-harmonics with $l=0,\pm1$. Of course, after substituting (16) to (13) and (15) we get a set of coupled equations: i.e., the equation for f_l contains the harmonics $f_{l\pm1}$ and $f_{l\pm2}$. However, for magnetized plasmas this coupling can be taken into account by the standard approximation using the small 'magnetization' parameter $r_\lambda/l_\perp<<1$, when the Larmor radius $r_\lambda=\sqrt{2TMc}/(eH_0)$ of charged particles is much less than the scale length $l_\perp$ of nonuniformity of the plasma-wave parameters in the direction perpendicular to $\mathbf{H}_0$. Thus, to evaluate the ε_{11}, ε_{12}, ε_{21}, ε_{22}, and ε_{33} dielectric tensor components, we should solve three equations for the first (f_0 and $f_{\pm1}$) harmonic of the perturbed distribution function. As a result, the 2D transverse and longitudinal current density components can be calculated as

$$j_1 = j_n = j_{(1)} + j_{(-1)}\,, \qquad j_2 = j_b = i[\,j_{(-1)} - j_{(1)}\,], \tag{17}$$

$$j_3 = j_{\|} = \sum_{\alpha}^{e,i_1,i_2,..} j_{\|,\alpha} = 2\pi \sum_{\alpha}^{e,i_1,i_2,..} e_\alpha \int_{-\infty}^{\infty} v_{\|} dv_{\|} \int_0^{\infty} f_{0,\alpha} v_{\perp} dv_{\perp}\,, \tag{18}$$

where

$$j_{(l)} = \sum_{\alpha}^{e,i_1,i_2,..} j_{(l)\alpha} = \pi \sum_{\alpha}^{e,i_1,i_2,..} e_\alpha \int_{-\infty}^{\infty} dv_{\|} \int_0^{\infty} f_{l,\alpha} v_{\perp}^2 dv_{\perp}\,, \quad l = \pm 1\,. \tag{19}$$

The simplest solution of the Vlasov equations (13) and (15) can be realized by introducing the new variables associated with the conservation integrals of the particle motion and the equation of the magnetic field lines for the concrete $\mathbf{H}_0$-field configuration. The conservation integrals (the motion invariants) are the same for any axisymmetric magnetized plasmas: $v_{\|}^2 + \mathrm{v}_{\perp}^2 = const$ and $\mathrm{v}_{\perp}^2 / H_0 = const$, as the conservation of particle energy and magnetic moment, respectively. Thus instead of $(v_{\|},\mathrm{v}_{\perp})$-variables one can introduce the $v = \sqrt{v_{\|}^2 + \mathrm{v}_{\perp}^2}$ and $\mu = \dfrac{\mathrm{v}_{\perp}^2}{v_{\|}^2 + \mathrm{v}_{\perp}^2}\dfrac{H_{0\min}}{H_0}$ variables. By the parameter μ, all plasma particles (of any kind) can be separated on the two groups of the trapped and untrapped particles. After this, the reduced Vlasov equations should be resolved separately for each group of particles to evaluate their contributions to the 2D perturbed current density components and, respectively, to the corresponding dielectric tensor elements.

CONCLUSION

In conclusion let us summarized the main results of the paper. In particular, there is derived, in the cylindrical coordinates, the linearized Vlasov equation (5) for the perturbed distribution functions of plasma particles which are confined in the arbitrary curvilinear magnetic field. It is shown that this equation is suitable to develop the kinetic wave theory in the such 2D axisymmetric plasma configurations as tokamaks, straight mirror traps and inner part of the planetary magnetospheres. The stationary magnetic fields in these models have one minimum in the equatorial plane. As a result, the charged particles (electrons, protons, heavy ions) should be separated on the magnetically trapped and untrapped particles. Such separation can be done by the parameter μ corresponding to a non-dimensional magnetic moment, analyzing the conditions when the parallel velocity of plasma particles is equal to zero.

It should be noted, since we find the perturbed distribution functions of the trapped and untrapped particles in the zero order of 'magnetization' parameters neglecting the drift effects, our dielectric characteristics in [3-6, 8, 9] are not taking into account the finite "beta" and finite Larmor radius corrections, and the finite banana widths of the trapped and untrapped particles. On the other hand, such an approach allows us to define the main contributions of the trapped and untrapped particles to the transverse and longitudinal dielectric tensor components in the simplest analytical form suitable to analyze the wave-particle interactions due to both the Cherenkov and fundamental ($l = \pm 1$) cyclotron resonances, accounting for the transit time and bounce resonances of the untrapped and trapped particles, respectively.

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