УДК 533.951

MEANDERING SHAPE OF DISPERSION CURVES OF MAGNETIZED PLASMA-FILLED WAVEGUIDES

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Received 11 October 2011

The article presents the results of theoretical research of meandering behavior of dispersion curves of waveguides with magnetized plasma filling. It is shown that regardless of plasma density joint solutions for different pairs of equations, satisfying the dispersion relation, can exist. It is determined that they are unique dispersion curves intersection points with domain boundaries which establish the solution set of these equations. It is shown that for each dispersion curve, belonging to one or several domains can be unambiguously determined. It is discovered that intersection of dispersion curve with the domain boundaries results in its meandering behavior. It is shown that in the absence of the points of such intersection the dispersion curve cannot leave the boundaries of one domain and in this case meandering behavior of the curve is absent.
KEY WORDS: plasma-filled waveguide, magnetized plasma, dispersion equation, dispersion curves, hybrid waves

ЗВИВИСТИЙ ВИГЛЯД ДИСПЕРСІЙНИХ КРИВИХ У МАГНІТОАКТИВНИХ ПЛАЗМОВИХ ХВИЛЕВОДАХ

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У статті представлено результати теоретичного дослідження звивистої поведінки дисперсійних кривих у хвилеводах із магнітоактивним плазмовим наповненням. Показано, що незалежно від густини плазми можуть існувати спільні розв’язки для різних пар рівнянь, які задовольняють дисперсійному співвідношенню. Встановлено, що вони є єдиними точками перетину дисперсійних кривих із межами областей, які формує множина усіх розв’язків цих рівнянь. Показано, що для кожної дисперсійної кривої можна однозначно встановити приналежність до однієї або декількох таких областей. Виявлено, що перетин дисперсійною кривою меж областей призводить до її звивистої поведінки. Показано, що за відсутності точок такого перетину дисперсійна крива не може покидати межи однієї області й у цьому випадку звивиста поведінка у кривої не спостерігається.
КЛЮЧОВІ СЛОВА: плазмовий хвилевод, магнітоактивна плазма, дисперсійне рівняння, дисперсійні криві, гібридні хвилі

ИЗВИЛИСТЫЙ ВИД ДИСПЕРСИОННЫХ КРИВЫХ В МАГНИТОАКТИВНЫХ ПЛАЗМЕННЫХ ВОЛНОВОДАХ

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В статье представлены результаты теоретического исследования извилистого поведения дисперсионных кривых в волноводах с магнитоактивным плазменным наполнением. Показано, что вне зависимости от плотности плазмы могут существовать совместные решения для различных пар уравнений, удовлетворяющие дисперсионному соотношению. Установлено, что они являются единственными точками пересечения дисперсионных кривых с границами областей, которые образует множество всех решений этих уравнений. Показано, что для каждой дисперсионный кривой можно однозначно установить принадлежность одной или нескольким таким областям. Обнаружено, что пересечение дисперсионной кривой границ областей приводит к ее извилистому поведению. Показано, что в отсутствии точек такого пересечения дисперсионная кривая не может покидать пределы одной области и в этом случае извилистое поведение у кривой не наблюдается.
КЛЮЧЕВЫЕ СЛОВА: плазменный волновод, магнитоактивная плазма, дисперсионное уравнение, дисперсионные кривые гибридные волны

Research of dispersion properties of plasma-filled systems attracted and still attracts the attention of the researchers. This is confirmed both by the list of publications made in the second half of the last century [1-8] and papers published relatively recently [9-13]. This topic is of scientific and practical interest for solving of a number of technological problems, such as: creation of new oscillators of high-power electromagnetic radiation; development of prospective devices for transportation of high-current beams of charged particles; search of effective plasma methods of charged particles acceleration and etc. The key task for studying of plasma-filled waveguide structures for the mentioned above applications is the studying of the case of smooth cylindrical waveguide, filled with “cold”, collisionless, homogeneous plasma, placed into the longitudinal magnetic field of finite strength. A deep and comprehensive analysis of this case is required for obtaining of basic knowledge which allows simplifying the studying of more complicated and realistic plasma-filled structures.

Despite the fact that dispersion equation for cylindrical waveguide with magnetized plasma filling was obtained rather long time ago [1], some properties of its solutions up to date remain unclear. It is explained by a complicated form of dispersion equation, which in general case can be solved only numerically. At the same time numerical analysis by itself does not provide exhaustive description of some peculiarities of dispersion properties of plasma waveguides.

In particular, these include the quaint meandering shape of waveguides dispersion curves [4]. This effect constitutes not only the self interest. It also makes the procedure of searching the numerical solutions of dispersion equation more complicated (see for example, Fig.7 from [13]).

Possible explanation of this effect was proposed in the paper [5], which shows the connection between meandering behavior of dispersion curves and opaque region, placed below the plasma frequency. This region is widening with increasing plasma density and thus according to [5] can displace and deform dispersion curves. At the same time it is known [11, 13] that even at low plasma densities, when the opaque region is almost absent, dispersion curves nevertheless can be essentially deformed compared to vacuum case. Therefore explanation of the meandering behavior of dispersion curves, proposed in [5], cannot be unique. Alternative reason of dispersion curves deformation was proposed in [11]. According to [11] deformation can be a result of the coupling between hybrid modes of plasma waveguide. The possibility of such coupling is mentioned in [6]. In [11] dispersion curves for two families of waves were obtained. The coupling between waves of different families becomes essential only near intersection of these curves. Here their "reconnection" takes place which results in meandering structure of dispersion curves of the plasma waveguide. At the same time representation of waveguide's eigenwaves in the form of two families of weakly coupled waves, proposed in [11], has the limited domain of applicability. It becomes useless for dense plasma.

The purpose of this work is the theoretical analyses of formation of meandering shape of dispersion curves under arbitrary plasma densities in waveguide. For this purpose together with numerical calculations a number of new analytical results are used.

PARTICULAR SOLUTIONS OF DISPERSION EQUATION FOR WAVEGUIDE FILLED WITH MAGNETIZED PLASMA

We consider a smooth cylindrical metallic waveguide completely filled with "cold" collisionless plasma with immobile ions. The waveguide is placed in the external magnetic field B_0 directed parallel to waveguide axis oz. Dispersion equation for such waveguide is well known and has been presented in the literature for several times [1-11]. Let us present it in the following way

$$D(\omega,k_z)=\alpha_2\Phi_l\left(k_2R\right)J_l\left(k_1R\right)-\alpha_1\Phi_l\left(k_1R\right)J_l\left(k_2R\right)=0\,, \tag{1}$$

where $\alpha_{1,2}=(k_zk\varepsilon_2)^2+\chi^2(\varepsilon_3\chi^2+\varepsilon_1k_{1,2}^2)$, $\Phi_l\left(k_iR\right)\equiv\Phi_l\left(\omega,k_z,R\right)=k_iRJ_l'\left(k_iR\right)-lbJ_l\left(k_iR\right)$, $b=\varepsilon_2k^2/\chi^2$, $2\varepsilon_1k_{1,2}^2=-(\varepsilon_1+\varepsilon_3)\chi^2-k^2\varepsilon_2^2\pm\sigma$, $\sigma=(\varepsilon_3-\varepsilon_1)\sqrt{(k_z^2-k^2)^2+4k^2k_z^2\varepsilon_3\,\omega^2/\omega_H^2}$, $\chi^2=k_z^2-\varepsilon_1k^2$, $\varepsilon_1=1-\omega_p^2/\left(\omega^2-\omega_H^2\right)$, $\varepsilon_2=-\omega_p^2\omega_H/\left(\omega(\omega^2-\omega_H^2)\right)$, $\varepsilon_3=1-\omega_p^2/\omega^2$, $\omega_p=\left(4\pi e^2n_e/m_e\right)^{1/2}$ - electron plasma frequency, $\omega_H=eB_0/m_ec$ - electron cyclotron frequency, e, m_e, n_e - charge, mass and density of plasma electrons, c - speed of light in vacuum, $k=\omega/c$ - wave vector of the free space, R - waveguide radius, k_z and l axial and azimuth wave numbers.

When deducing the equation (1) it is assumed that in cylindrical coordinates $\{r,\ \varphi,z\}$ in linear approximation perturbations for waveguide's eigenfields, density and velocity of plasma electrons have a form $A(\mathbf{r},t)=A(r)\exp(-i\omega t+ik_zz+il\varphi)$.

Let us find some partial solutions of the equation (1) and determine the conditions of their existence. We assume that $\alpha_{1,2}\neq 0$. From (1), we see that if for some s and n there is a joint solution of equations

$$k_1\left(\omega,k_z\right)=\mu_{l,s}/R\,, \tag{2}$$

$$k_2\left(\omega,k_z\right)=\mu_{l,n}/R\,, \tag{3}$$

then it is also a solution of dispersion equation (1). Here $\mu_{l,s}$ – is the s root of the lth-order Bessel function. Such solutions of dispersion equation will be marked as (k_{z0},ω_0).

Solutions of combined equations such as

$$k_1\left(\omega,k_z\right)=\gamma_{l,s}/R\,, \tag{4}$$

$$k_2\left(\omega,k_z\right)=\gamma_{l,n}/R\,, \tag{5}$$

for different s and n are also belong to particular solutions of dispersion equation (1), which are marked as (k_{z0}',ω_0'). Here $\gamma_{l,s}$, $s=1,2,...$ are zeros of the function $\Phi_l\left(x\right)$. These zeros are placed between the neighbor roots of the Bessel function $J_l(x)$ [14]. In general case they depend on ω and k_z, since $b=b\left(\omega,k_z\right)$. Exception is the case $l=0$, when $\Phi_0\left(x\right)=-xJ_1\left(x\right)$. In this case - $\gamma_{0,s}=\mu_{1,s}$.

System of equations (2), (3) and (4), (5) could have solutions only when k_1 and k_2 are real. This is due to the fact that zeros of the functions $J_l(x)$ and $\Phi_l\left(x\right)$ are real. The exclusion can be only for the first root of the function $\Phi_l\left(x\right)$, which becomes purely imaginary one under the condition $lb<-1$ [14]. However for generality we will exclude it from consideration, assuming condition $\mu_{l,1} < \gamma_{l,1} < \mu_{l,2}$ to be fulfilled. Thus all particular solutions (k_{z0}, ω_0) and (k'_{z0}, ω'_0) of dispersion equation (1) correspond to the volume waves and belong to the zones of (ω, k_z) plane in Fig.1 bounded by bold line.

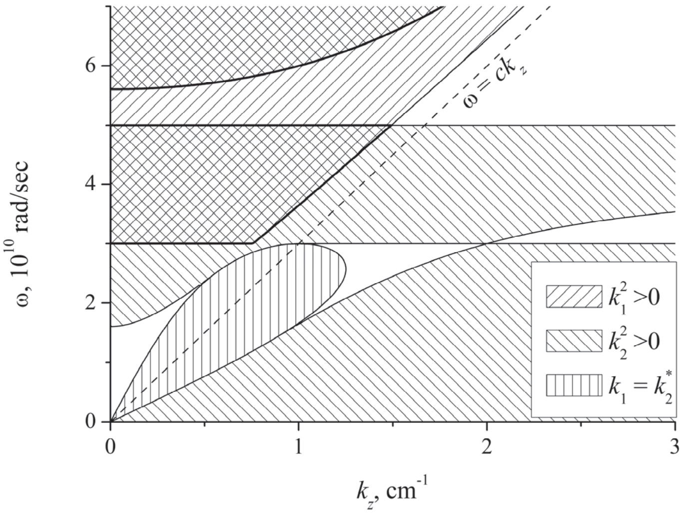

Fig.1. Domains of existence of different types of waves of plasma waveguide at $\omega_p = 3\times10^{10}$ rad/sec, $\omega_H = 4\times10^{10}$ rad/sec

We will consider only the zone, placed below the upper hybrid frequency $\omega_1 = \sqrt{\omega_H^2 + \omega_p^2}$ and will find all s and n for which combined equations (2) and (3) have solutions in this zone.

For this purpose we consider the curves in the (ω, k_z) plane formed by the solutions of the equations (2) and (3). These curves behaviorally similar to dispersion curves for two families of weakly coupled waves in [11] (see Fig. 1a in [11]). Solutions of the equation (3) for $n = 1, 2...$ establish within the frequency interval $\left(\min\left(\omega_p, \omega_H\right) < \omega < \omega_1\right)$ the infinite set of curves with asymptotics $\max\left(\omega_p, \omega_H\right)$ at $k_z \to \infty$. Curve described by equation (2) at fixed s resembles parabola with vertex at $k_z = 0$ and asymptotics $k = k_z$ at $k_z \to \infty$.

Thus such parabola has the infinite number of points of intersection with curves, satisfying the equation (3) for different n, if it gets into the frequency interval $\left(\min\left(\omega_p, \omega_H\right) < \omega < \omega_1\right)$. This condition can be presented in the following way

$$\mu_{l,s} < \omega_H R / c . \tag{6}$$

Condition (6) is fulfilled when the cut-off frequency of the $TM_{1,s}$ – mode is located below the upper hybrid frequency (see, for example, [10]).

Knowing the peculiarities of behavior for curves in the (ω, k_z) plane, satisfying the equations (2) and (3), it is easy to determine n such that these equations have joint solutions. If for the specified s condition (6) is fulfilled, than such joint solutions exist for all n, for which roots of equation (3) at $k_z = 0$ exceeds the cut-off frequency of the $TM_{1,s}$ – mode. This condition can be presented in the following way

$$2\left(\frac{\mu_{l,s}c}{R}\right)^2 < \omega_H^2 + \left(\frac{\mu_{l,n}c}{R}\right)^2 - \sqrt{\left[2\omega_p^2 + \omega_H^2 + \left(\frac{\mu_{l,n}c}{R}\right)^2\right]^2 - 4\left[\omega_p^4 + \left(\frac{\mu_{l,n}c}{R}\right)^2 \omega_1^2\right]} . \tag{7}$$

For the case of rare plasma, when $2\omega_p^2 << \omega^2, \omega_H^2$, it reduces to the form $n > s$. Thus system of equations (2) and (3) has the solutions for all s and n, satisfying the conditions (6) and (7). It is clear that such solutions in the frequency region below the upper hybrid frequency are absent, when $\mu_{l,1} > \omega_H R / c$.

In order to find the solutions of system of equations (2) and (3) it is convenient to reduce it to one equation for ω

$$\varepsilon_2^2 k^2 + \varepsilon_1 \frac{\mu_{l,n}^2 + \mu_{l,s}^2}{R^2} - sign\left(\frac{\omega^2 - \omega_1^2}{(\omega - \omega_H)(\omega - \omega_p)}\right)\left(\varepsilon_1 + \varepsilon_3\right)\left(\varepsilon_2^2 k^4 + \frac{\varepsilon_1}{\varepsilon_3}\frac{\mu_{l,n}^2 \mu_{l,s}^2}{R^4}\right)^{1/2} = 0 . \tag{8}$$

Frequency ω_0 is the solution of equation (8) for given s and n. When the value of ω_0 is defined, one can find k_{z0} using relation [12]

$$k_{z0} = \left.\left(\frac{\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_1\varepsilon_3}{\varepsilon_1 + \varepsilon_3} k^2 - \frac{\varepsilon_1 (\mu_{l,n}^2 + \mu_{l,s}^2)}{\left(\varepsilon_1 + \varepsilon_3\right) R^2}\right)^{1/2}\right|_{\omega = \omega_0} . \tag{9}$$

Since the sets of equations (2), (3) and (4), (5) have similar forms, the obtained results can also be applied to the combined equation (4), (5) using the change $\mu_{l,m} \to \gamma_{l,m}$ ($m = s, n$).

Solutions (k_{z0}, ω_0) are the unique solutions of equations (2) and (3) for different s and n, satisfying the dispersion equation (1). This is due to the fact that the roots of the equation $xJ_l'(x) - lbJ_l(x) = 0$ at arbitrary finite b are placed between the zero of Bessel function $\mu_{l,s}$, where $s = 1,2,...$ [14]. That is why the inequality $\Phi_l(\mu_{l,s}) \neq 0$ holds. Similarly it can be shown that among all solutions of equations (4) and (5) for different s and n, only solutions (k_{z0}', ω_0') can satisfy the dispersion equation. This implyies that at $\alpha_{1,2} \neq 0$, dispersion curves of the plasma waveguide in the (ω, k_z) plane are not able to cross any solution of equations (2)-(5) except points (k_{z0}, ω_0) and (k_{z0}', ω_0').

Thus solutions of these equations for different s and n create in the (ω, k_z) plane the boundaries of domains, containing the dispersion curves of the plasma waveguide.

MEANDERING SHAPE OF DISPERSION CURVES IN THE WAVEGUIDES FILLED WITH MAGNETIZED PLASMA

It is known that in the waveguide filled with magnetized plasma waves of the TE and TM types are coupled. Exception is the case $k_z = 0$ when α_1 identically goes into zero*. In this case dispersion equation (1) is divided into two independent equations (2) and (5) for $s, n = 1,2...$. Their solutions at $k_z = 0$ correspond to cut-off frequencies of $TM_{l,s}$ and $TE_{l,n}$ waves [6]. Solutions of these equations also describe at small k_z the behavior of dispersion curves for two families of weakly coupled waves studied at [11]. Using classification [6], the hybrid waves of plasma waveguide will be indicated as EH and HE waves. At $k_z = 0$ EH and HE waves have TM and TE polarizations correspondingly.

Below in calculations, the frequencies $\omega_p = 3\times10^{10}$ rad/sec and $\omega_H = 4\times10^{10}$ rad/sec is assumed to be fixed. We will study only the frequency region $\omega < \omega_1$. It is known [11], the deformation effect of dispersion curves is distinct here.

Let us build in (ω, k_z) plane the boundaries of domains, containing the dispersion curves of the plasma waveguide.

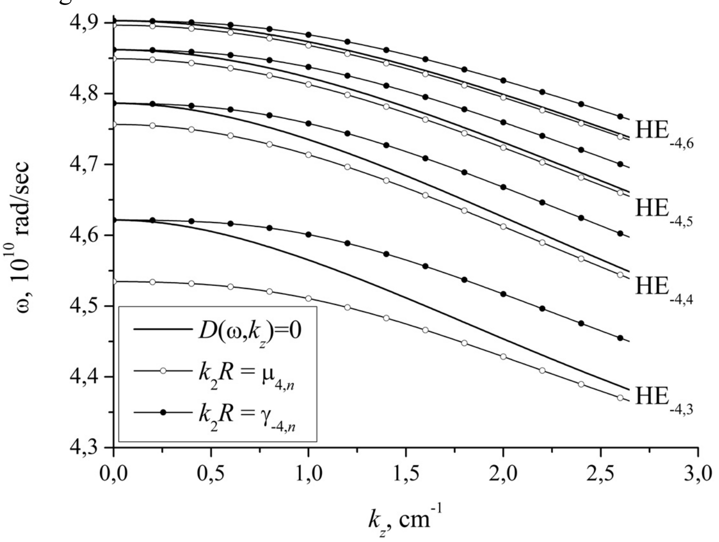

Fig.2. Solutions of equations (1), (2), and (5) at $l = -4$, $R = 5$ cm.

Under the condition $\mu_{l,1} > \omega_H R / c$ such boundaries are formed by the solutions of equation (3) and (5) for $n = 1,2...$ (Fig.2), since in the frequency region $\omega < \omega_1$ equations (2) and (4) cannot have solution for $s = 1,2...$. As a consequence, points (k_{z0}, ω_0) and (k_{z0}', ω_0') cannot exist here. If points (k_{z0}, ω_0) and (k_{z0}', ω_0') are absent then dispersion curves cannot cross the domain boundaries and each of them remains within one domain. As it can be seen from Fig.2 in this case meandering behavior of dispersion curves is absent.

When the dispersion curves contain points (k_{z0}, ω_0) and (k_{z0}', ω_0'), the boundaries of domains change significantly. The boundaries take the form of cells of two different meshes (Fig.3). Boundaries of the first mesh are formed by solutions of equations (2) and (3) for all s and n, satisfying the conditions (6) and (7), and of the second by solutions of equations (4) and (5). The unique points of intersection of these boundaries by dispersion curves are mesh nodes – the points (k_{z0}, ω_0) and (k_{z0}', ω_0'). This constrains the behavior of dispersion curves, which becomes meandering (Fig.3). The effect becomes more evident (Fig. 4) with the increase of number of Bessel function roots $\mu_{l,s}$, satisfying the condition (6), and simultaneous growth of density of distribution of points (k_{z0}, ω_0) and (k_{z0}', ω_0') in the (ω, k_z) plane. Since condition (6) does not include the plasma density we can come to the conclusion: despite the fact that meandering shape of dispersion curves is a purely plasma effect it can be noticeable at arbitrarily small (but nonzero) plasma density. This conclusion is in accordance with the results [11, 13].

Plasma density is also not included into condition $\mu_{l,1} > \omega_H R / c$ at which the meandering shape of dispersion

* Nevertheless coupling between TE and TM azimuthal waves at $k_z = 0$ can be realized in the case of current-carrying plasma [15].

curves is absent. Thus if for chosen values of waveguide radius R and magnetic field B_0 this condition is fulfilled, the deformation effect for dispersion curves within the frequency range $\omega < \omega_1$ will be negligibly small for all plasma densities in the waveguide.

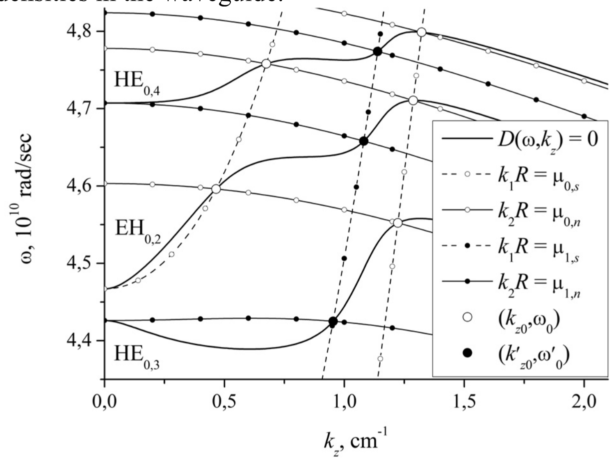

Fig.3. Solutions of equations (1)-(5), and also points (k_{z0}, ω_0), (k'_{z0}, ω'_0) ($l = 0$, $R = 5$ cm).

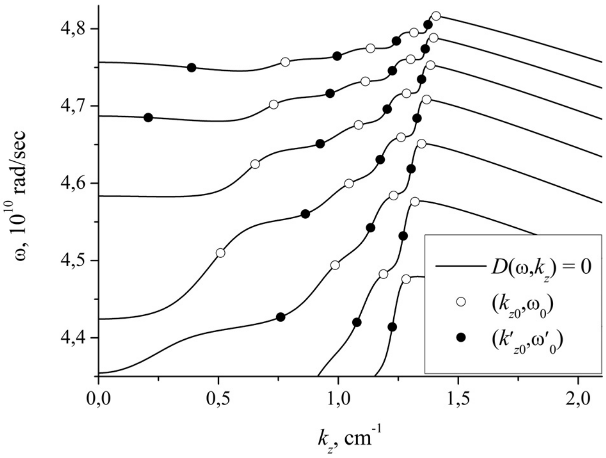

Fig.4. Solution of dispersion equation (1) and points (k_{z0}, ω_0), (k'_{z0}, ω'_0) at $l = 0$, $R = 11.2$ cm.

All mentioned above is true both for right-polarized ($l > 0$) and for left- polarized ($l < 0$) modes of the plasma waveguide. It is known [8, 10] that in general case dispersion properties of this modes are different. However, existence and location of the points (k_{z0}, ω_0) in the (ω, k_z) plane do not depend on the sign of azimuth index l. It is explained by the fact that such dependence in equations (2) and (3) is absent, since $\mu_{-l,s} = \mu_{l,s}$. Therefore, in the (ω, k_z) plane points (k_{z0}, ω_0) are points of mating of dispersion curves for right- polarized and left- polarized modes (Fig.5). Such mating was observed in [10].

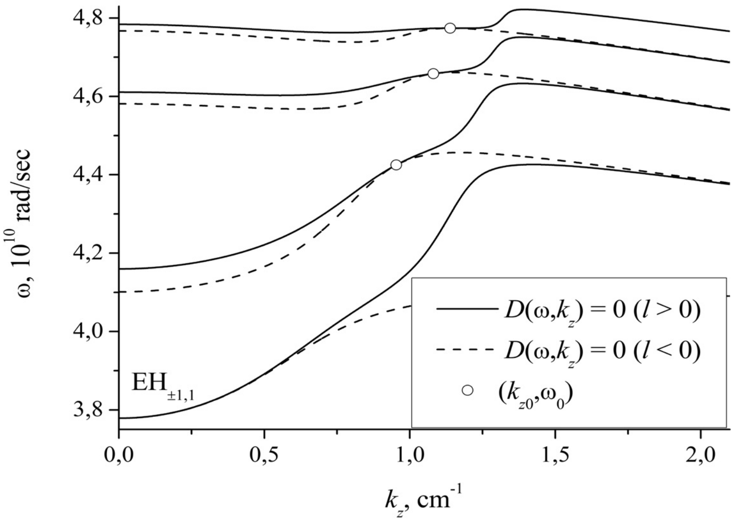

Fig.5. Dispersion curves of the right- polarized ($l > 0$) and left-polarized ($l < 0$) modes, and also their mating points ($|l| = 1$, $R = 5$ cm).

DOMAINS OF EXISTENCE FOR DISPERSION CURVES OF MAGNETIZED PLASMA-FILLED WAVEGUIDE

Belonging to one or several domains bounded by the solutions of equations (2)-(5) for different s and n can be determined in advance for the mode with specified radial index. For this purpose one should know mode cut-off frequency [6] and dispersion curve behavior in the range of $k_z << k$ [11].

If at higher k_z, solutions of equations (2)-(5) create the boundaries of mesh with nodes (k_{z0}, ω_0) and (k'_{z0}, ω'_0) (Fig.6), one should know the slopes of the tangents to the dispersion curves at the points (k_{z0}, ω_0) and (k'_{z0}, ω'_0) to determine mesh nodes and cells to which points of the dispersion curve belong.

The slope of the tangent to the dispersion curve at one of the points (k_{z0}, ω_0) can be found, taking into account that in small neighborhood of this point

$$k_1 = \mu_{l,s} / R + \Delta k_1 , \tag{10}$$

$$k_2 = \mu_{l,n} / R + \Delta k_2 , \tag{11}$$

and $|\Delta k_{1,2}| R << 1$. Then from dispersion equation (1) we find

$$\Delta k_2 = \left. \frac{\mu_{l,n}\alpha_2}{\mu_{l,s}\alpha_1} \right|_{(k_{z0},\omega_0)} \Delta k_1 . \tag{12}$$

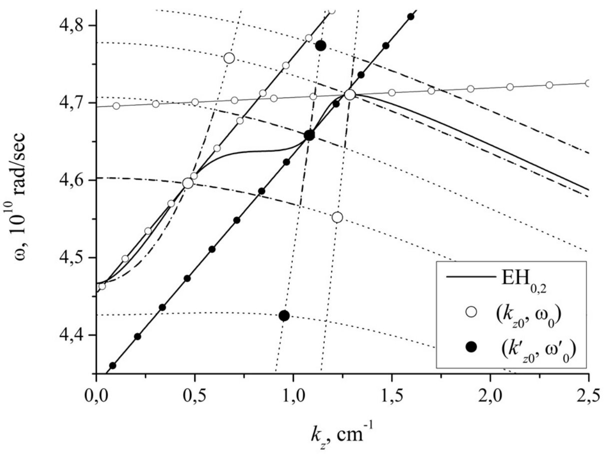

Fig.6. The same as in Fig.3, and also tangents to the dispersion curve of the $EH_{0,2}$ mode at the points (k_{z0},ω_0), (k'_{z0},ω'_0).

Equations (10)-(12) constitute parametric form of the equation of the straight line in the plane of variables k_1 and k_2. For traditional variables $k_z = k_{z0} + \Delta k_z$ and $\omega = \omega_0 + \Delta\omega$ this straight line has a form of tangent to the dispersion curve at the point (k_{z0},ω_0). To build it in (ω, k_z) plane one should take into account that in the small neighborhood of this point

$$\Delta k_{1,2} = \Delta k_z \left.\frac{\partial k_{1,2}}{\partial k_z}\right|_{(k_{z0},\omega_0)} + \Delta\omega \left.\frac{\partial k_{1,2}}{\partial \omega}\right|_{(k_{z0},\omega_0)} .$$

In a similar way equation of tangent to the dispersion curve at one of the points (k'_{z0},ω'_0) can be found. In this case relation between k_1 and k_2 is determined by equations

$$k_1 = \gamma_{l,s} / R + \Delta k_1 , \tag{13}$$

$$k_2 = \gamma_{l,n} / R + \Delta k_2 , \tag{14}$$

$$\Delta k_2 = \left.\frac{\alpha_1 \Phi'_l(\gamma_{l,s}) J_l(\gamma_{l,n})}{\alpha_2 \Phi'_l(\gamma_{l,n}) J_l(\gamma_{l,s})}\right|_{(k'_{z0},\omega'_0)} \Delta k_1 , \tag{15}$$

where $\Phi'_l(x) = d\Phi_l / dx$ at $b = b(k'_{z0},\omega'_0) = const$.

If dispersion curve belongs to the zone of the (ω, k_z) plane in which the condition $k_i^2 R^2 < \mu_{l,1}^2$ is fulfilled ($i = 1$ or 2) (Fig.2), then with increasing k_z such curve is not able to leave the interval between the neighbor solutions of equations $k_j R = \mu_{l,n}$ and $k_j R = \gamma_{l,n}$ ($j = 2$ or 1).In particular, this is valid for the dispersion curves of pseudo-surface waves with $k_i^2 < 0$, $k_j^2 > 0$ (see Fig.1).

Thus for each mode, boundaries of one or several domains in which points of dispersion curves are located, can be determined in advance. These boundaries define the frequency intervals for all k_z in which solutions of equation (1) for the selected mode are located.

Knowledge of these intervals for different modes allows researching numerically dispersion properties of one or several selected modes, establishing for the specified k_z minimal frequency resolution required for searching the neighbor solutions of equation (1), and with further change of k_z avoiding jump from someone definite solution to another one.

These issues are urgent in case when the studied frequency spectrum is dense. In the magnetized plasma-filled waveguide, dispersion curves of hybrid mode thicken to the frequency ω_1 when k_z is finite and to the frequencies ω_p and ω_H when $k_z \to \infty$.

CONCLUSIONS

The work presents theoretical analyses of the meandering behavior of dispersion curves of waveguides with magnetized plasma filling and conditions of its appearance. This effect is mostly evident within the frequency range below the upper hybrid frequency ω_1.

Analysis is conducted taking into account the knowledge of some particular solutions of dispersion equation for plasma waveguide.

It is shown that in order to find particular solutions it is convenient to introduce instead of variables ω and k_z new variables – transverse wave numbers k_1 and k_2. It can be shown that, in particular, solutions of combined equations $k_1 R = \mu_{l,s}$ and $k_2 R = \mu_{l,n}$ at different s and n satisfy the dispersion equation. It is determined that within the frequencies range $\omega < \omega_1$, such solutions exist for all n, satisfying the condition (7) and s, for which root $\mu_{l,s}$ of the lth-order Bessel function, waveguide radius R and electron cyclotron frequency ω_H satisfy the condition $\mu_{l,s} < \omega_H R / c$. It is shown that joint solutions of equations $k_1 R = \mu_{l,s}$ and $k_2 R = \mu_{l,n}$ for different s and n, marked in the paper (k_{z0},ω_0) are unique solutions of these equations which satisfy the dispersion equation for the plasma waveguide. In the (ω, k_z) plane they are mating points of dispersion curves for right- polarized $(l > 0)$ and left-polarized $(l > 0)$ modes. Solutions of combined equations $k_1R = \gamma_{l,s}$ and $k_2R = \gamma_{l,n}$ for different s and n are located on the dispersion curves between the neighbor points (k_{z0}, ω_0). Here $\gamma_{l,s}$ - are zeros of the function $\Phi_l(x)$ (see dispersion equation (1)). It is shown that except for points (k'_{z0}, ω'_0) in the (ω, k_z) plane none of the solutions of equations $k_1R = \gamma_{l,s}$ and $k_2R = \gamma_{l,n}$ satisfy the dispersion equation for magnetized plasma-filled waveguide.

It is found that solutions of equations $k_1R = \mu_{l,s}$ and $k_2R = \mu_{l,n}$ for s, satisfying the condition $\mu_{l,s} < \omega_H R / c$ and n satisfying the condition (7) establish the mesh in the (ω, k_z) plane. Similar mesh is formed by solutions of equations $k_1R = \gamma_{l,s}$ and $k_2R = \gamma_{l,n}$.

As it goes from the obtained results, the dispersion curves of the plasma-filled waveguide cannot cross the boundaries of these meshes nowhere except their nodes (points (k_{z0}, ω_0) and (k'_{z0}, ω'_0) in the (ω, k_z) plane).

It is shown that this constrains the behavior of dispersion curves which take of the meandering form. Moreover the effect becomes stronger, when the density of distribution of points (k_{z0}, ω_0) and (k'_{z0}, ω'_0) in the (ω, k_z) plane becomes higher and disappears in the absence of these points. It is shown the last takes place on condition that $\mu_{l,1} > \omega_H R / c$, which does not depend on plasma density. This implies that plasma density does not play dominating role in the deformation of dispersion curves of the magnetoactive plasma waveguide. When the condition $\mu_{l,1} > \omega_H R / c$ is fulfilled, regardless of plasma density all dispersion curves cannot leave the intervals between the solutions of equations $k_2R = \mu_{l,n}$ and $k_2R = \gamma_{l,n}$ for $n = 1, 2...$ In the reverse case the meandering shape of dispersion curves can be noticeable even at small plasma densities in the waveguide.

It is shown that any point of dispersion curve belongs either to the node or to the cell of one of two meshes. Such belonging can be unambiguously determined knowing the slopes of the tangents to the dispersion curves at the points (k_{z0}, ω_0) and (k'_{z0}, ω'_0). Thus for each point of dispersion curve, belonging to the certain frequency interval can be determined in advance which allows simplifying the procedure of numerical search of solution of dispersion equation.

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