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THE MODEL OF REALISTIC PHOTOMULTIPLIER RESPONSE

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It is considered model of a realistic photomultiplier (PM), allowing get by the producing function method in enough general event a new analytical expression for its response in broad range of the light intensity. Seven free parameters of the model have a clear physical interpretation. Under stable equipment working and big statistics aproximating the PM response functions, got from model by expressions, has much get good convergence that enables to receive the single electron parameters, gain, the average number of photoelectrons, noise with good accuracy. The results are used in the study of long-term stability of the scintillation muon counters for the CDF II detector (Fermilab).
KEY WORDS: photomultiplier, Poisson distribution, Gauss distribution, background charge distribution, producing function, indicative function, average number photoelectrons, gain factor of the photomultiplier.

МОДЕЛЬ ВІДГУКУ РЕАЛЬНОГО ФОТОПОМНОЖУВАЧА

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Розглядається модель реального фотопомножувача (ФЭУ), що дозволяє отримати методом виробляючих функцій, в досить загальному випадку новий аналітичний вираз для його відгуку в широкому діапазоні засвічень. Сім вільних параметрів моделі мають ясний фізичний сенс. При стабільній роботі апаратури і великій статистиці апроксимація спектру, отриманого виразами з моделі, має дуже хорошу збіжність, що дає можливість отримати параметри одноелектронного піку, середнє число фотоелектронів, коефіцієнт посилення, параметри шумів з високою точністю. Отримані результати використовуються для дослідження довготривалої стабільності сцинтіляційних мюонних лічильників CDF II (Фермілаб).
КЛЮЧОВІ СЛОВА: фотопомножувач, розподіл Пуассона, розподіл Гаусса, одно електронний розподіл, виробляюча функція, характеристична функція, середнє число фотоелектронів, коефіцієнт посилення фотопомножувача.

МОДЕЛЬ ОТКЛИКА РЕАЛЬНОГО ФОТОУМНОЖИТЕЛЯ

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Рассматривается модель реального фотоумножителя (ФЭУ), позволяющая получить методом производящих функций в достаточно общем случае новое аналитическое выражение для его отклика в широком диапазоне засветок. Семь свободных параметров модели имеют ясный физический смысл. При стабильной работе аппаратуры и большой статистике аппроксимация спектра, полученными из модели выражениями, имеет очень хорошую сходимость, что даёт возможность получить параметры одноэлектронного пика, среднее число фотоэлектронов, коэффициент усиления, параметры шумов с высокой точностью. Полученные результаты используются для исследования долговременной стабильности сцинтилляционных мюонных счетчиков установки CDF II (Фермилаб).
КЛЮЧЕВЫЕ СЛОВА: фотоумножитель, распределение Пуассона, распределение Гаусса, одноэлектронное распределение, производящая функция, характеристическая функция, среднее число фотоэлектронов, коэффициент усиления фотоумножителя.

When work with photosensor (the photomultiplier, hybrid, semiconductor photomultiplier, avalanche photo diodes etc.) happens to deal with measurement small level signal at a rate of single electron level [1-8]. After the ADC calibration from analysis of the spectrum is extracted information on average count of photoelectrons.

This three different methods is done [1]:

a) In the event of clearly visible single photoelectrons, this peak is used for direct calibration of the ADC scale in numbers of photoelectrons. Then average number of photoelectrons is got from the formula of the gravity center spectrum distribution:

$$\mu = \frac{1}{\Delta} \cdot \left\{ \frac{\sum_i A_i \cdot N_i}{\sum_i N_i} - N_{ped} \right\}, \quad (1)$$

where μ - an average number of photoelectrons, N_i - i-channel of the ADC, A_i - a number impulses in i-channel, N_{ped} - a pedestal, Δ - a calibrate factor, equal difference of the position of photoelectrons and the pedestal.

A single photoelectrons peak well stands out at a rate of noise only for it is enough narrow class of the photomultiplier, so quantacon named.

b) In the event of small number photoelectrons (≤ 5÷8) and expecting Poisson their distribution the estimation numbers of photoelectrons possible to do on inefficiency P(0,μ), i.e. from the fraction of triggers in the pedestal region:

$$\mu = -\ln \cdot P(0;\mu)\,, \tag{2}$$

where P(0,μ) is found directly from the charge spectrum:

$$P(0;\mu) = \frac{N_{ped}}{N}\,. \tag{3}$$

c) In the event of small number photoelectrons (≥ 5÷8) and expecting Gauss their distribution estimation of the photoelectrons number possible to do on formula:

$$\mu = \left(\frac{2.36 \cdot A}{FWHM}\right)^2, \tag{4}$$

where A - a center gravity of the spectrum, FWHM – a full width half-maximum of the Gauss distribution.

The results of these methods of estimation μ will agree between themselves at level 10÷100% and conducting calibration is significantly determined by conditions. Usually this divergence is explained by the correctness of the assumptions about the nature of distributions at different stages of the formation of photoelectrons and in the processes of further multiplication.

However, in high energy physics in the development and optimization design of plastic scintillation detectors required method of precise determination the light yield and the technical attenuation length (TAL). To do this it need to know the method to find the average number of photoelectrons with high accuracy, especially at low illumination. At first it's been done in [2-4] by means of computer analysis of the amplitude spectrum of the photomultiplier.

The aim of this work is developed the model of the photomultiplier by the method of generating functions. The response to the small lighting it is possible to accurately restore after of the approximation of amplitude spectrum by the function, which is obtained from the natural physical assumptions about the nature of the distributions of the number of photoelectrons on the photo-cathode, the dynodes, and also noise pulses. We will follow, in essence, the approaches, developed in the works [1-8], and also use its designations.

In this work the new general expression for the charge distribution of the spectrum is obtained analytically. With grow good convergence according to the method the chi-square it can be obtain with the high accuracy basic parameters for FEU-85 photomultiplier. The results are used by the author for the study of long-term stability the test sample of the scintillation muon counter CDF II Detector.

PROCESSES OF PHOTOCONVERSION AND COLLECTION OF THE PHOTOELECTRONS

Let us examine of the PM photo cathode by the flash of the light by the constant amplitude (light-emitting diode or scintillator) luminous source with the average number of formation of photons in each flash, equal to m. The probability of the appearance n photons in the flash is subordinated, as is known, to Poisson's law (5):

$$P(n;m) = \frac{m^n \cdot e^{-m}}{n!}\,. \tag{5}$$

Let us assume that all photons consecutively fall on photoelectric cathode. This can be made, considering that the photons are distributed in the time also according to the Poisson's low. In other words, the probability of hit of two photons simultaneously is extremely low, which correctly for the wide range of the illumination is carried out, as a rule, in the majority of the cases. Natural to call this assumption the approximation of linear model. Then the process of electron escape with the probability p is possible due to the photo effect from the photo cathode. Accordingly the probability of the absence of the electron let designate q = 1 – p. Thus, this process is considered as binary and it is described by Bernoulli's law:

$$P([0,1]) = p \cdot q\,. \tag{6}$$

Further it is well known that the convolution of distributions (5) and (6) also it is the Poisson distribution, but with the average value of the photoelectrons:

$$\mu = m \cdot \varepsilon\,, \tag{7}$$

where ε – the average value of the photoelectrons for one photon with the sufficiently large statistics, is a constant value for this type of photo cathode, the known as quantum efficiency of photo cathode. Then the escape probability n of electrons with an average quantity of photoelectrons μ is equal:

$$P(n;\mu) = \frac{\mu^n \cdot e^{-\mu}}{n!}. \tag{8}$$

Hence is obtained expression (2) estimate of the magnitude μ with n = 0. For the low level of inherent noise and small intensity of the light source this formula it can be used for calibrating of the ADC scale or obtaining μ directly from the spectrum. Let us note that μ it is the parameter, which characterizes not only light source, but also the quantum efficiency of photo cathode and the efficiency of the collection of photoelectrons to the first dynode.

THE PROCESS OF THE MULTIPLICATION

The charge distribution of one-electron peak $G_1(x)$ at the output of photomultiplier (the response PM to the single photoelectron) we will approximate by Gaussian distribution:

$$G_1(x) = \frac{1}{\sigma_1\sqrt{2\pi}} \cdot \exp\left(-\frac{(x-Q_1)^2}{2\sigma_1^2} \right), \tag{9}$$

where Q_1, σ_1 - respectively the center of gravity (the average charge position of peak on the ADC scale) and the standard deviation of one-electron peak. In this case it is assumed that there are no losses of electrons on the dynodes in the process of multiplication. Let us also calculate characteristic function for (9) according to the definition:

$$\varphi_\xi^1(t) = Me^{i\cdot t\cdot x} = \exp(i \cdot Q_1 \cdot t - \tfrac{1}{2} \cdot \sigma_1^2 \cdot t^2). \tag{10}$$

The PM output of the one-electron peak is connected with the multiplication factor according to the formula:

$$Q_1 = e \cdot g, \tag{11}$$

where e - electron charge in the picocoulombs. Value Q_1 is determined also in the picocoulombs, if is used by QDC converter.

For n of photoelectrons the formula (9) is generalized with the aid of the simple multiplication of the functions $\varphi_\xi^1(t)$:

$$G_n(x) = \frac{1}{\sigma_1\sqrt{2\pi \cdot n}} \cdot \exp\left(-\frac{(x-nQ_1)^2}{2n\sigma_1^2} \right). \tag{12}$$

It is here important to note that the expression (12) has a limit $G_0(x) = \delta(x)$ with n → 0, that can be understood as the absence of the output signal of photomultiplier, if there are no photoelectrons.

Now, following the theorem about the total probability, it is possible to obtain the charge spectrum of photomultiplier, summarizing the appropriate partial contributions:

$$S^{ideal}(x) = \sum_{n=0}^{\infty} S_n(x) = P(0,\mu)\cdot G_0(x) + P(1,\mu)\cdot G_1(x) + ... + P(n,\mu)\cdot G_n(x) + ... =$$
$$\sum_{n=0}^{\infty} \frac{\mu^n \cdot e^{-\mu}}{n!} \cdot \frac{1}{\sigma_1\sqrt{2\pi \cdot n}} \cdot e^{-\frac{(x-n\cdot Q_1)^2}{2\cdot n\sigma_1^2}}, \tag{13}$$

and also the characteristic function of this spectrum:

$$\varphi_\xi^{ideal}(t) = \sum_{n=0}^{\infty} \frac{\mu^n \cdot e^{-\mu}}{n!} \cdot e^{i\cdot n\cdot Q_1\cdot t - \frac{1}{2}\cdot n\cdot \sigma_1 \cdot t^2}, \tag{14}$$

which we will call ideal, since the noise of photomultiplier was not considered.

NOISES OF THE PHOTOMULTIPLIER

In the real photomultiplier with the processes of photo-conversion and sequential multiplication of charge by dynode system is present the noise of different nature, which indicates the generation of the additional charge. Such noise can cause current in the PM anode circuit even in the absence light of photo cathode. The sources of noise are sufficiently well studied (the autoemmision from the electrodes, the thermoemission, photon and ionic feedback, internal and out radioactivity, the leakage currents so forth), although strictly in the construction of concrete photomultiplier they sufficiently individual and technologically not so it is simple to decrease them.

It is now important to note that from the point of view of the model, all sources of PM noise can be divided into two types:

$$B(x) = B_1(x) + B_2(x), \qquad \int_{-\infty}^{\infty} B(x)\cdot dx = 1. \tag{15}$$

The first type - the so-called "pedestal", which is determined by the nontrivial probability of the absence of the electrons with the presence of the trigger (the "gate" signal). Pedestal is, thus, the noise within the limits of the window of trigger signal. Let us represent pedestal by normal distribution with the center of gravity Q_0 and deviation of σ_0:

$$B_1(x) = \frac{1-w}{\sigma_0\sqrt{2\pi}}\cdot e^{-\frac{(x-Q_0)^2}{2\cdot\sigma_0^2}}. \tag{16}$$

The procedure of the standardization (16) gives the weighting factor 1- w of a portion of this type of noise in (15). The characteristic function for B1(x) will be equal:

$$\varphi_\xi^{B_1}(t) = M\cdot e^{i\cdot t\cdot x} = (1-w)\cdot e^{i\cdot Q_0\cdot t - \frac{1}{2}\cdot\sigma_0^2\cdot t^2}. \tag{17}$$

The second type of noise depends from the source strength of the light and it is statistically connected with the processes the propagation of photoelectrons in the photomultiplier. Therefore one should expect that their distribution is subordinated to exponential function and lies more to the right pedestal, since charges less than "the zero" be cannot.

$$B_2(x) = w\cdot\alpha\cdot\theta(x)\cdot e^{-\alpha\cdot(x-Q_0)}. \tag{18}$$

The index of exponential curve α, which characterizes this type of noise, and the contribution w to the total balance accordingly (15) enter in (18) for guaranteeing the procedure of standardization. Function $\theta(x-Q_0)$ shows that "negative" charges in the spectrum be there must not. The corresponding characteristic function for $B_2(x)$ will be equal:

$$\varphi_\xi^{B_2}(t) = M\cdot e^{i\cdot t\cdot x} = \frac{e^{\alpha\cdot Q_0}}{\alpha - i\cdot t}. \tag{19}$$

THE NEW FORMULA OF THE CHARGE PM OUTPUT

The function of the approximation of real spectrum can be obtained by the convolution of ideal spectrum (13) and noise spectrums (16) and (18), which in the space of characteristic functions will be considered as the multiplication of the corresponding characteristic functions (14) and (17), (19):

$$\varphi_\xi^{real}(t) = \varphi_\xi^{real1}(t) + \varphi_\xi^{real2}(t) = \varphi_\xi^{ideal}(t)\cdot\varphi_\xi^{B_1}(t) + \varphi_\xi^{ideal}(t)\cdot\varphi_\xi^{B_2}(t). \tag{20}$$

Then inverse transformation will give the function for the approximation of the real spectrum of the photomultiplier:

$$S^{real}(x) = S^{real1}(x) + S^{real2}(x) =$$
$$\sum_{n=0}^{\infty}\frac{\mu^n\cdot e^{-\mu}}{n!}\cdot\left\{\frac{1-w}{\sigma_n\cdot\sqrt{2\cdot\pi}}\cdot e^{-\frac{(x-Q_n)^2}{2\cdot\sigma_n^2}} + \alpha\cdot w\cdot\theta(x-Q_0)\cdot e^{-\alpha\cdot(x-Q_n)+\frac{1}{2}\cdot n\cdot\sigma_1^2\cdot\alpha^2}\right\}, \tag{21}$$

where the following parameters of spectrum are determined as:

$$Q_n = Q_0 + n\cdot Q_1,\ \sigma_n = \sqrt{\sigma_0^2 + n\sigma_1^2} \approx \begin{cases}\sigma_0 & n=0,\\ \sqrt{n}\cdot\sigma_1 & n>0.\end{cases} \tag{22}$$

The formula (21) has seven free parameters: Q_0 and σ_0 describe pedestal, Q_1 and σ_1 – the one-electron peak, α and w - second type noise and their weight contribution to general noise accordingly (15), μ - average number of photoelectrons. With n → 0 it is reduced actually to (2) and (3). The integral of the pedestal will be equal N_{ped}. Approximation for large n will take the form (4).

AN EXPERIMENTAL STUDY OF PM OUTPUT

Some functions from the base approximation (21) was used for work with PM FEU-85. Measuring circuit was selected analogous which was used in the work [2]. As the light source was used the light-emitting diode (LED), which given the pulses with the duration of 10 ns from the fast generator. LED was at a distance of 30 cm from the photocathode. Analog signal from PM was measured by LeCroy 2249A QDC and further through the CAMAC controller CC02 [9] was introduced into the computer. The signal from the fast generator fed to the shaper to generate a signal «Gate» QDC.

Some typical experimental spectrum of single-electron peak are shown in Fig.1. with helping ROOT5 [10]. For a satisfactory convergence of the method of chi-squared enough of the first some Poisson distributions.

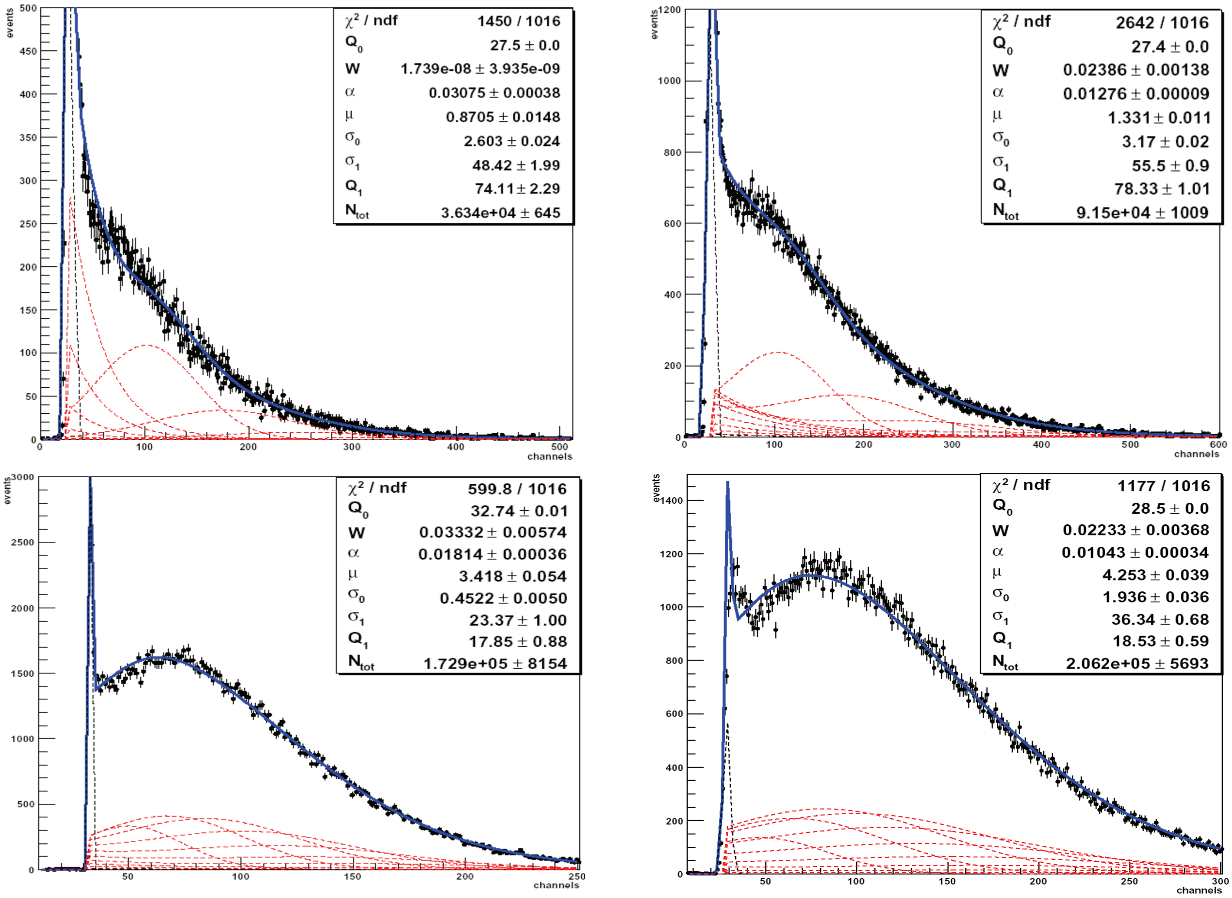

Fig.1. Typical LED spectrums from PM FEU-85.

CONCLUSION

The model of the response of real photomultiplier on the basis of the method of characteristic functions is developed. The new general formula, which describes the function of the response of photomultiplier, is obtained with the parameters, which make clear and physical sense. The criterion of the applicability of the functions of the approximation of charge spectrum is convergence on the chi-square test. This approach was used with studies of the long-term stability of the parameters of the scintillation muon detectors of the CDF II (Fermilab), which requires a separate publication. The author hopes that this approach may also be useful in the recovery of the photon distribution of the light source.

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